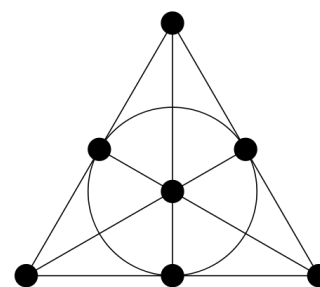


My research interests are centered at the intersection of three areas of combinatorics: design theory, coding theory, and finite geometries. My dissertation, “Applications of finite geometries to designs and codes,” addresses fundamental questions about these structures, including existence, uniqueness, and growth. By understanding the structure and power of these designs and codes, I am also able to produce excellent results in other fields, such as quantum error-correcting codes.

A  $t$ - $(v, k, \lambda)$  *design* is a highly structured combinatorial object which consists of a set of *points* and a collection of  $k$ -subsets of points called *blocks*. Every  $t$ -subset of points must appear in exactly  $\lambda$  blocks. The “geometric designs” – constructed from the points and  $d$ -dimensional subspaces of a finite geometries (such as a projective or affine geometry) – are common examples of designs. The incidence matrix of a design gives rise to an *error-correcting code*: a vector subspace which inherits many of the properties of the design. The famous “Fano plane” is an example of a design derived from a finite geometry.



Designs and codes provide excellent opportunities to “hook” undergraduate students into research. These closely related structures can be introduced in the context of vector spaces, finite fields, geometries, graph theory, information theory, abstract algebra, and many other topics of interest to upper-level students. Students interested in fields as broad as voting theory, geometry, deep-space communications, or biological field experiments may be naturally drawn to designs and codes.

The motivating problem in my dissertation is Hamada’s conjecture, which proposes that the incidence matrix of each geometric design has the unique minimal  $p$ -rank among all designs with the same parameters. Hamada’s conjecture was inspired by the study of codes which admit *majority-logic decoding*, a particularly simple and easy to implement decoding method. Indeed, Hamada’s conjecture implies that the finite geometry codes are among those best-suited for such decoding.

In general, Hamada’s conjecture is an open question. The conjecture is known to be true for certain families of parameters, but there are also sporadic counterexamples. Jungnickel and Tonchev [JT09] found the first infinite family of counterexamples to Hamada’s conjecture, by “distorting” the blocks of certain projective geometry designs. I proved a corresponding result for affine geometry designs, also producing the first infinite family in this case [CJT11]. I generalized this construction to produce an exponentially growing number of non-isomorphic designs with affine parameters, the first time that such a result has been shown for affine geometry designs [CJT10]. I also proved that the “distorted” designs (both affine and projective) retain many of the best properties of the geometric designs.

The next natural steps in this area have to do with classification. My previous work opens many questions, which I plan to continue pursuing: In which cases do counterexamples exist? What is special about those parameter sets? Because Hamada’s conjecture is known to be *true* in many cases, there is some distinction or method of classification which we have not yet discovered. All known counterexamples share most of the best properties of geometric designs. Is this true in general? Recent work of Jungnickel and Tonchev [JT] shows how a generalized concept of “dimension” given powerful results. Many of these questions are well-suited to undergraduate research. Students

may study the structure of known counterexamples, which encourages a deeper understanding of fundamental combinatorial concepts. The generalized idea of “dimension” is brand new, and many results are still waiting to be discovered. Undergraduates interested in mathematical programming can search for new counterexamples, or exhaustively enumerate designs for small parameters.

In addition, designs form an excellent basis for better understanding seemingly unrelated fields of study. In particular, I have studied several applications of designs to the construction and analysis of quantum error-correcting codes. *Entanglement-assisted quantum error-correcting codes* (or EAQECCs) are a large new category of quantum codes. They are especially interesting, because they provide a way to construct quantum codes from *any* classical code. Previous work generally produced examples of EAQECCs through random search, or by hand construction. Using the structure of designs, I helped to provide the first general method which organized and systematized the creation of EAQECCs with specified properties [FCV<sup>+</sup>10]. There are also many other types of quantum codes, most of which can be constructed in highly combinatorial ways. As with EAQECCs, the parameters and properties of these codes may be determined from the original finite geometries. For example, I have constructed infinite families of  $q$ -ary quantum codes from designs related to finite geometries [CT]. These provide an ever-expanding range of opportunities for further research.

The study of quantum information theory – specifically quantum error-correcting codes – is growing quickly. New code constructions appear frequently. Many of these constructions are based around assumptions or requirements which are fundamentally combinatorial. By understanding the power and structure of designs, codes, and finite geometries, I am in a position to develop excellent new examples of quantum codes in a rapidly expanding field. The links between discrete mathematics and quantum codes give undergraduate researchers an opportunity to contribute to a cutting-edge and exciting field of study.

- [CJT10] D. Clark, D. Jungnickel, and V. D. Tonchev, *Exponential bounds on the number of designs with affine parameters*, Journal of Combinatorial Designs **18** (2010), 475–487.
- [CJT11] ———, *Affine geometry designs, polarities, and Hamada’s conjecture*, Journal of Combinatorial Theory, Series A **118** (2011), 231–239.
- [CT] D. Clark and V. D. Tonchev, *Nonbinary quantum codes derived from finite geometries*, Finite Fields and their Applications, to appear.
- [FCV<sup>+</sup>10] Y. Fujiwara, D. Clark, P. Vandendriessche, M. De Baeck, and V. D. Tonchev, *Entanglement-assisted quantum low-density parity-check codes*, Physical Review A **82** (2010), 1–19.
- [JT] D. Jungnickel and V. D. Tonchev, *A Hamada type characterization of the classical geometric designs*, Designs, Codes, and Cryptography, to appear.
- [JT09] ———, *Polarities, quasi-symmetric designs, and Hamada’s conjecture*, Designs, Codes, and Cryptography **51** (2009), 131–140.