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Calculus 1 Project 1
Logarithmic Differentiation

• For $y = x^x$, can we say that $\frac{dy}{dx} = x.x^{x-1} = x^x$? Why?

• For $y = x^x$, can we say that $\frac{dy}{dx} = \ln(x).x^x$? Why?

• Now use the implicit differentiation in Maple to find $\frac{dy}{dx}$

It seems that Maple was able to give us a "nice" answer, but the question is: how can we find the derivative of this function?

Let us now explore this method (Logarithmic Differentiation):

• Introduce the natural logarithm to both sides of the equation $y = x^x$.(type in your answer).

• Simplify the previous step using the properties of the natural log in your book on page 25.

• Can we find

- $\frac{dy}{dx}$ now using <u>implicit differentiation</u>? If yes, then **type** in your answer **<u>without</u>** using Maple.

- Does your answer match Maple's answer in the second part? What needs to be done to get Maple's answer?

In the previous exercise we saw that the Logarithmic Differentiation method is powerful when the base and the exponent are both functions of x.

- Can we use the same method to find $\frac{dy}{dx}$ in this case $y = \frac{x^4.e^x(3x+2)}{6^{\ln x}(5-x)^2}$? If yes, find the derrivative and check your answer with Maple.

First we introduce the natural log to both sides of the equation to get

$$\ln(y) = \ln\left(\frac{x^4.e^x(3x+2)}{6^{\ln x}(5-x)^2}\right)$$

$$\ln(y) = \ln\left(x^4.e^x(3x+2)\right) - \ln\left(6^{\ln x}(5-x)^2\right)$$
$$\ln(y) = \ln\left(x^4\right) + \ln\left(e^x\right) + \ln(3x+2) - \ln\left(6^{\ln x}\right) - \ln\left((5-x)^2\right)$$
$$\ln(y) = 4\ln(x) + x + \ln(3x+2) - \ln(6)\ln x - 2\ln(5-x)$$

Now we take the implicit derivative of the whole equation with respect to x to get

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{4}{x} + 1 + \frac{3}{3x+2} - \frac{\ln(6)}{x} + \frac{2}{5-x}$$

Multiplying both sides by y gives

$$\frac{dy}{dx} = y.\left(\frac{4}{x} + 1 + \frac{3}{3x+2} - \frac{\ln(6)}{x} + \frac{2}{5-x}\right)$$

$$\frac{dy}{dx} = \frac{x^4 . e^x \cdot (3x+2)}{6^{\ln x} \cdot (5-x)^2} \cdot \left(\frac{4}{x} + 1 + \frac{3}{3x+2} - \frac{\ln(6)}{x} + \frac{2}{5-x} \right)$$

$$restart : y = \frac{x^4 \cdot e^x \cdot (3x+2)}{6^{\ln(x)} \cdot (5-x)^2}$$

$$y = \frac{x^4 e^x (3x+2)}{6^{\ln(x)} (5-x)^2} \quad (1)$$

$$\xrightarrow{\text{implicit differentiation}}$$

$$-\frac{x^3 e^x \left(81x + 4x^2 + 40 - 3x^3 - 13\ln(6)x + 3\ln(6)x^2 - 10\ln(6)\right)}{6^{\ln(x)} (-5+x)^3} \quad (2)$$

- Now find $\frac{dy}{dx}$ in each of the following cases. You may choose any method you want. Show all your work, and check your answer with Maple.
- $y = \frac{(\ln x)^x}{3^{2x-1}}$

- $y = x^{\ln x} . (\tan x)^{\sqrt{x}}$

Can you conclude in which cases the Logarithmic Differentiation should be used? Give two cases, and write them in your own words.