Back to the Basics: Revisiting the Development Accounting Methodology

Online Appendix

A1. Decomposing the variation in observables into the variation attributable to each observable component $(obs_{n,i})$.

Recall from the main text that the variance of observables can be expressed as $\text{var}[observables] = \sum_n \text{var}[obs_{n,i}] + 2\sum_n \sum_m \text{cov}[obs_{n,i}, obs_{m,i}]$. Each observable component is given by $obs_{n,i} = \frac{1}{2}(s_{n,i} + \overline{s_n})(\ln x_{n,i} - \overline{\ln x_n})$ where x_n represents factor of production *n* relative to raw labor, s_n is the corresponding factor share and a bar above a variable indicates the mean of that variable across all countries in the sample.

Two factor scenario

Suppose there are only two factors of production, *n* and *m*. Denote $\rho_{obs_{n,i},obs_{m,i}}$ as the statistical correlation between $obs_{n,i}$ and $obs_{m,i}$. If all of the correlation between $obs_{n,i}$ and $obs_{m,i}$ is attributed to $obs_{n,i}$, the relative variances can be computed according to the following decomposition:

$$\frac{\left(1-\rho^2_{obs_{n,i},obs_{m,i}}\right)\text{var}[obs_{m,i}]}{\text{var}[observables]} + \frac{\left\{sd[obs_{n,i}] + sd[obs_{m,i}]\rho_{obs_{n,i},obs_{m,i}}\right\}^2}{\text{var}[observables]} = 1. \qquad \text{(A1)}$$

The variation in observables attributable to variation in $obs_{m,i}$ is represented by the first term on the left hand side of equation (A1). The second term represents the variation in observables attributable to variation in $obs_{n,i}$.

Alternatively, all correlation between $obs_{n,i}$ and $obs_{m,i}$ can be attributed to $obs_{m,i}$, in which case the relative variance decomposition takes the form:

$$\frac{\left\{sd\left[obs_{m,i}\right]+sd\left[obs_{n,i}\right]\rho_{obs_{n,i},obs_{m,i}}\right\}^2}{\mathrm{var}\left[observable\ s\right]}+\frac{\left(1-\rho^2_{obs_{n,i},obs_{m,i}}\right)\mathrm{var}\left[obs_{n,i}\right]}{\mathrm{var}\left[observable\ s\right]}=1. \tag{A2}$$

As in equation (A1), the first and second terms in equation (A2) can be interpreted as the fractions of variation in observables attributable to $obs_{m,i}$ and $obs_{n,i}$, respectively.

Once the variance decompositions in equations (A1) and (A2) have been computed, an upper and lower bound for the variation in observables accruing to $obs_{n,i}$ and $obs_{m,i}$ can be determined.

Three factor scenario

Suppose that the subscript *n* from equation (7) runs over three factors of production. The methodology holds for any three factors, but for purposes of illustration, I will assume that the three factors are physical capital per worker *k*, effective labor per worker *h* and natural capital per worker *z*.

Equation (14) can be expressed as

$$\begin{aligned}\mathrm{var}\left[observables\right]=\mathrm{var}\left[obs_{k,i}\right]+\mathrm{var}\left[obs_{h,i}\right]+\mathrm{var}\left[obs_{z,i}\right]+2\,\mathrm{cov}\left[obs_{k,i},obs_{h,i}\right]\\+2\,\mathrm{cov}\left[obs_{k,i},obs_{z,i}\right]+2\,\mathrm{cov}\left[obs_{h,i},obs_{z,i}\right].\end{aligned} \tag{A3}$$

There are three covariances, and for each covariance, there are two extreme allocations of the correlation between the two observable components. All of the correlation can be attributed to one component or the other. Define the six extreme allocations as follows:

a = all $\rho_{obs_{k,i},obs_{h,i}}$ attributed to $obs_{k,i}$

b = all $\rho_{obs_{k,i},obs_{z,i}}$ attributed to $obs_{k,i}$

c = all $\rho_{obs_{h,i},obs_{z,i}}$ attributed to $obs_{h,i}$

d = all $\rho_{obs_{k,i},obs_{h,i}}$ attributed to $obs_{h,i}$

e = all $\rho_{obs_{k,i},obs_{z,i}}$ attributed to $obs_{z,i}$

f = all $\rho_{obs_{h,i},obs_{z,i}}$ attributed to $obs_{z,i}$

There are eight combinations of these allocations that are possible. Given a-f above, these combinations are: (a,b,c); (a,b,f); (a,e,c); (a,e,f); (d,b,c); (d,b,f); (d,e,c); and (d,e,f). I follow the general decomposition described above for the two factor scenario to estimate the relative variances so that combination (a,b,c) yields:

$$\frac{\operatorname{var}[obs_{k,i}] + 2\operatorname{cov}[obs_{k,i}, obs_{h,i}] + 2\operatorname{cov}[obs_{k,i}, obs_{z,i}] + \operatorname{var}[obs_{h,i}]\rho^2_{obs_{k,i},obs_{h,i}} + \operatorname{var}[obs_{z,i}]\rho^2_{obs_{k,i},obs_{z,i}}}{\operatorname{var}[observables]} + \frac{\left(1-\rho^2_{obs_{k,i},obs_{h,i}}\right)\operatorname{var}[obs_{h,i}] + 2\operatorname{cov}[obs_{h,i}, obs_{z,i}] + \operatorname{var}[obs_{z,i}]\rho^2_{obs_{h,i},obs_{z,i}}}{\operatorname{var}[observables]} + \frac{\left(1-\rho^2_{obs_{h,i},obs_{z,i}} - \rho^2_{obs_{k,i},obs_{z,i}}\right)\operatorname{var}[obs_{z,i}]}{\operatorname{var}[observables]} = 1 \tag{A4}$$

The first, second and third terms on the left hand side of equation (A4) correspond to the variation in observables attributable to variation in $obs_{k,i}$, $obs_{h,i}$ and $obs_{z,i}$, respectively.

As a further illustration, the relative variance decomposition corresponding to combination (d,b,f) is.

$$\frac{\left(1-\rho^2_{obs_{k,i},obs_{h,i}}\right)\operatorname{var}[obs_{k,i}] + 2\operatorname{cov}[obs_{k,i}, obs_{z,i}] + \operatorname{var}[obs_{z,i}]\rho^2_{obs_{k,i},obs_{z,i}}}{\operatorname{var}[observables]} + \frac{\left(1-\rho^2_{obs_{h,i},obs_{z,i}}\right)\operatorname{var}[obs_{h,i}] + 2\operatorname{cov}[obs_{k,i}, obs_{h,i}] + \operatorname{var}[obs_{k,i}]\rho^2_{obs_{k,i},obs_{h,i}}}{\operatorname{var}[observables]} + \frac{\left(1-\rho^2_{obs_{k,i},obs_{z,i}}\right)\operatorname{var}[obs_{z,i}] + 2\operatorname{cov}[obs_{h,i}, obs_{z,i}] + \operatorname{var}[obs_{h,i}]\rho^2_{obs_{h,i},obs_{z,i}}}{\operatorname{var}[observables]} = 1 \tag{A5}$$

As in equation (A4), the first, second and third terms on the left hand side of equation (A5) are interpreted as the variation in observables attributable to variation in $obs_{k,i}$, $obs_{h,i}$ and $obs_{z,i}$, respectively.

After all eight variance decompositions have been computed, an upper and lower bound for the variation in observables accruing to each of the three observable components can be determined.

A2. Decomposing the variation in each observable component $(obs_{n,i})$ into the variation attributable to $\psi_{n,i}$ and $\phi_{n,i}$, the share and factor portions of $obs_{n,i}$, respectively.

In the first decomposition I assume that all interaction between $\psi_{n,i}$ and $\phi_{n,i}$ reflects variability in $\psi_{n,i}$. The relative variance decomposition is given by

$$\frac{E^2(\phi_{n,i})\mathrm{var}[\psi_{n,i}] + Interaction_{\psi_{n,i},\phi_{n,i}} + E^2(\psi_{n,i})\mathrm{var}[\phi_{n,i}]\rho^2_{\psi_{n,i},\phi_{n,i}}}{\mathrm{var}[obs_{n,i}]} + \frac{(1-\rho^2_{\psi_{n,i},\phi_{n,i}})E^2(\psi_{n,i})\mathrm{var}[\phi_{n,i}]}{\mathrm{var}[obs_{n,i}]} = 1 \quad \text{(A6)}$$

where

$$\begin{aligned} Interaction_{\psi_{n,i},\phi_{n,i}} = {} & E\left[(\Delta\psi_{n,i})^2(\Delta\phi_{n,i})^2\right] + 2E(\psi_{n,i})E\left[(\Delta\psi_{n,i})(\Delta\phi_{n,i})^2\right] + 2E(\phi_{n,i})E\left[(\Delta\phi_{n,i})(\Delta\psi_{n,i})^2\right] \\ & + 2E(\psi_{n,i})E(\phi_{n,i})\mathrm{cov}[\psi_{n,i},\phi_{n,i}] - \mathrm{cov}^2[\psi_{n,i},\phi_{n,i}] \end{aligned}$$

and $\rho_{\psi_{n,i},\phi_{n,i}}$ denotes the statistical correlation between $\psi_{n,i}$ and $\phi_{n,i}$. The first term on the left hand side of equation (A6) represents the fraction of variation in $obs_{n,i}$ attributable to variation in $\psi_{n,i}$. The second term represents the fraction of variation attributable to $\phi_{n,i}$.

Alternatively, if all of the interaction is assumed to reflect variability in $\phi_{n,i}$, the relative variances can be estimated according to

$$\frac{(1-\rho^2_{\psi_{n,i},\phi_{n,i}})E^2(\phi_{n,i})\mathrm{var}[\psi_{n,i}]}{\mathrm{var}[obs_{n,i}]} + \frac{E^2(\psi_{n,i})\mathrm{var}[\phi_{n,i}] + Interaction_{\psi_{n,i},\phi_{n,i}} + E^2(\phi_{n,i})\mathrm{var}[\psi_{n,i}]\rho^2_{\psi_{n,i},\phi_{n,i}}}{\mathrm{var}[obs_{n,i}]} = 1. \quad \text{(A7)}$$

As in equation (A6), the first term on the left hand side of equation (A7) is the fraction of variation in $obs_{n,i}$ attributable to variation in $\psi_{n,i}$, and the second term is the fraction of variation in $obs_{n,i}$ attributable to variation in $\phi_{n,i}$.

A3. Constructing the factor share estimates

Factor share estimates are constructed in accordance with Sturgill (2012). Total labor's share is computed as

$$\eta+\beta=\frac{Employee\ Compensation}{GDP-Indirect\ Taxes-Gross\ Mixed\ Income}\quad , \tag{A8}$$

and total capital's share is the perfect competition counterpart and given by

$$\alpha+\gamma=1-\left(\frac{Employee\ Compensation}{GDP-Indirect\ Taxes-Gross\ Mixed\ Income}\right)\ . \tag{A9}$$

Recall from section 2.2 that α , β , η and γ are the elasticities of output with respect to physical capital, human capital, raw labor and natural capital, respectively. The elasticity of output with respect to a factor is equal to the factor share only if markets are perfectly competitive. Thus, the implicit assumption in equations (A8) and (A9) is that factor shares measured using national income account data are reasonable estimates of aggregate output elasticities. This is standard in the literature as the values, typically constant parameters, which are routinely inserted for output elasticities in development and growth accounting, are really estimates of factor shares.

Total capital and total labor shares are computed for the year 2000. I obtain data for *Employee Compensation*, *GDP* and *Indirect Taxes* from table 4.1 of the 2009 version of the United Nations Yearbook of National Account Statistics (United Nations, 2010). Sturgill (2012) obtains these data from the 2007 UN yearbook. Each UN yearbook reports all available data for the previous ten or eleven years, and data for all years are updated with each publication. Therefore, the data from the 2009 yearbook reflect updated values for the year 2000.

Data for *Gross Mixed Income*, when they are available, are also obtained from table 4.1. For some countries, the value of *Gross Mixed Income* is included in the reported value of *Gross Operating Surplus*. In these cases total shares are estimated using equations (A8) and (A9) with *Imputed Gross Mixed Income* (*IGMI*) substituted for actual *Gross Mixed Income*. *Imputed Gross Mixed Income* is constructed by multiplying the share of self-employed persons in total employment by private sector income as follows:

$$IGMI=\left(\frac{Total\ Self\text{-}Employed\ Workers+Contributing\ Family\ Workers}{Total\ Employment-Not\ Classified}\right)\left(\begin{matrix}Gross\ Operating\ Surplus\ +\\ Employee\ Compensation\end{matrix}\right). \tag{A10}$$

Data for *Total Employment* and all of its components, including *Wage and Salaried Workers, Total Self-Employed Workers, Contributing Family Workers* and *Not Classified* are obtained from the KILM database (KILMnet, 2013).[1] If *Gross Mixed Income* must be imputed,[2] only countries for which self-employment as a fraction of total employment is less than or equal to 0.5 are included.

The estimates of total capital's share and total labor's share are presented in Tables A1 and A2, respectively.

Sturgill (2012), which follows Caselli and Feyrer (2007), shows that, under the assumption that physical and natural capital pay the same return, physical capital's share can be determined according to

$$\alpha = \frac{K}{C} \cdot (\alpha + \gamma) \tag{A11}$$

where $C = K+Z$ is the value of the *total capital stock*. Physical capital's share is proportional to the ratio of physical capital to total capital. Given my estimates of K, Z and $\alpha + \gamma$, estimates of α are obtained in accordance with equation (A11). These estimates are presented in Table A3.

In like manner, natural capital's share can be expressed as

$$\gamma = \frac{Z}{C} \cdot (\alpha + \gamma) \quad , \tag{A12}$$

and given estimates of total capital's share and physical capital's share, γ can equivalently be backed out as a residual. Table A4 reports my estimates of γ.

Human capital's share is estimated using returns to education and the percentage of the population in various educational attainment categories. As with the specification of human

[1] I use the sixth edition of the KILM database. The seventh edition is the most recent version but the data for the year 2000 are the same. The only difference between the sixth edition and the seventh edition is that the seventh edition includes *Contributing Family Workers* in the reported value for *Total Self Employed*.

[2] Seventeen of the 53 countries in my sample require *Imputed Gross Mixed Income*. They include Bolivia, Costa Rica, Denmark, Israel, Jamaica, Korea, Morocco, Namibia, Netherlands, New Zealand, Panama, Philippines, Romania, Russia, Sri Lanka, Trinidad and Tobago, and Tunisia.

capital in section 2.2, each year of schooling in each country is assumed to yield an 11.7% rate of return per year for the first four years, a 9.7% rate of return per year for the next four years, and a 7.5% rate of return per year for schooling beyond eight years. The percentage of the population aged 15 and over in seven educational attainment categories for the year 2000 is obtained for each country from Barro and Lee (2010). The categories include *No Schooling*, *Incomplete primary*, *Complete Primary*, *Incomplete Secondary*, *Complete Secondary*, *Incomplete Higher* and *Complete Higher*. These categories correspond to 0, 4, 8, 10, 12, 14 and 16 years of schooling, respectively. The returns to education imply a wage relative to no schooling for each educational attainment category. For example, workers with *Incomplete Higher* education would earn $1.117^4 \times 1.097^4 \times 1.075^6 = 3.48$ times as much as workers with *No Schooling*.

As in Sturgill (2012), which follows Pritchett (2001), the fraction of wages accruing to human capital is computed as

$$Human\ Capital's\ Share\ of\ Wages = \frac{\sum_{g=0}^{6}\left(w_g - w_0\right)\tau_g}{100 + \sum_{g=0}^{6}\left(w_g - w_0\right)\tau_g} \tag{A13}$$

where g indexes the seven educational attainment categories, w_g is the wage relative to no schooling, and τ_g is the percentage of a country's population in each educational attainment category. The numerator in equation (A13) represents total wages paid to human capital and the denominator represents total wages paid in the economy. The 100 in the denominator is the normalized value of total wages paid to raw labor; 100% of workers receive the relative wage of 1 for remuneration of raw labor.

Estimates of human capital's share of income are computed by multiplying total labor's share of income by *Human Capital's Share of Wages*. These estimates of β are reported in Table A5. Given estimates of total labor's share and human capital's share, raw labor's share can be computed as a residual. Estimates of η are reported in Table A6.

Table A1: Total Capital's Share, 2000

Country	Total Capital's Share	Country	Total Capital's Share
Argentina	0.529	Korea, Rep.	0.230
Australia	0.380	Latvia	0.463
Austria	0.344	Mexico	0.546
Belgium	0.350	Moldova	0.470
Bolivia	0.156	Morocco	0.413
Botswana	0.668	Mozambique	0.596
Brazil	0.457	Namibia	0.303
Bulgaria	0.548	Netherlands	0.359
Canada	0.387	New Zealand	0.395
Chile	0.481	Niger	0.369
Colombia	0.466	Norway	0.461
Costa Rica	0.285	Panama	0.529
Cote d'Ivore	0.491	Philippines	0.439
Denmark	0.326	Portugal	0.299
Egypt, Arab Rep.	0.557	Romania	0.214
Estonia	0.448	Russian Fedaration	0.461
Finland	0.427	Spain	0.336
France	0.347	Sri Lanka	0.079
Germany	0.328	Sweden	0.303
Greece	0.475	Switzerland	0.303
Honduras	0.411	Trinidad and Tobago	0.485
Hungary	0.392	Tunisia	0.392
Iran	0.647	United Kingdom	0.327
Israel	0.312	United States	0.278
Italy	0.443	Uruguay	0.420
Jamaica	0.115	Venezuela, RB	0.583
Japan	0.371		

Sources: Sturgill (2012) and author's calculations.

Table A2: Total Labor's Share, 2000

Country	Total Labor's Share	Country	Total Labor's Share
Argentina	0.471	Korea, Rep.	0.770
Australia	0.620	Latvia	0.537
Austria	0.656	Mexico	0.454
Belgium	0.650	Moldova	0.530
Bolivia	0.844	Morocco	0.587
Botswana	0.332	Mozambique	0.404
Brazil	0.543	Namibia	0.697
Bulgaria	0.452	Netherlands	0.641
Canada	0.613	New Zealand	0.605
Chile	0.519	Niger	0.631
Colombia	0.534	Norway	0.539
Costa Rica	0.715	Panama	0.471
Cote d'Ivore	0.509	Philippines	0.561
Denmark	0.674	Portugal	0.701
Egypt, Arab Rep.	0.443	Romania	0.786
Estonia	0.552	Russian Fedaration	0.539
Finland	0.573	Spain	0.664
France	0.653	Sri Lanka	0.921
Germany	0.672	Sweden	0.697
Greece	0.525	Switzerland	0.697
Honduras	0.589	Trinidad and Tobago	0.515
Hungary	0.608	Tunisia	0.608
Iran	0.353	United Kingdom	0.673
Israel	0.688	United States	0.722
Italy	0.557	Uruguay	0.580
Jamaica	0.885	Venezuela, RB	0.417
Japan	0.629		

Sources: Sturgill (2012) and author's calculations.

Table A3: Physical Capital's Share, 2000

Country	Physical Capital's Share	Country	Physical Capital's Share
Argentina	0.277	Korea, Rep.	0.175
Australia	0.216	Latvia	0.262
Austria	0.252	Mexico	0.304
Belgium	0.269	Moldova	0.216
Bolivia	0.038	Morocco	0.227
Botswana	0.397	Mozambique	0.149
Brazil	0.217	Namibia	0.172
Bulgaria	0.268	Netherlands	0.261
Canada	0.190	New Zealand	0.145
Chile	0.192	Niger	0.038
Colombia	0.160	Norway	0.255
Costa Rica	0.114	Panama	0.293
Cote d'Ivore	0.096	Philippines	0.224
Denmark	0.229	Portugal	0.216
Egypt, Arab Rep.	0.245	Romania	0.113
Estonia	0.270	Russian Fedaration	0.177
Finland	0.290	Spain	0.244
France	0.252	Sri Lanka	0.049
Germany	0.248	Sweden	0.215
Greece	0.331	Switzerland	0.231
Honduras	0.167	Trinidad and Tobago	0.125
Hungary	0.239	Tunisia	0.194
Iran	0.100	United Kingdom	0.234
Israel	0.231	United States	0.189
Italy	0.328	Uruguay	0.182
Jamaica	0.074	Venezuela, RB	0.157
Japan	0.296		

Sources: Sturgill (2012) and author's calculations.

Table A4: Natural Capital's Share, 2000

Country	Natural Capital's Share	Country	Natural Capital's Share
Argentina	0.252	Korea, Rep.	0.056
Australia	0.163	Latvia	0.200
Austria	0.091	Mexico	0.242
Belgium	0.081	Moldova	0.254
Bolivia	0.117	Morocco	0.186
Botswana	0.271	Mozambique	0.447
Brazil	0.240	Namibia	0.131
Bulgaria	0.280	Netherlands	0.098
Canada	0.197	New Zealand	0.250
Chile	0.290	Niger	0.331
Colombia	0.306	Norway	0.206
Costa Rica	0.171	Panama	0.237
Cote d'Ivore	0.395	Philippines	0.215
Denmark	0.097	Portugal	0.083
Egypt, Arab Rep.	0.312	Romania	0.101
Estonia	0.178	Russian Fedaration	0.284
Finland	0.137	Spain	0.092
France	0.095	Sri Lanka	0.030
Germany	0.079	Sweden	0.088
Greece	0.144	Switzerland	0.072
Honduras	0.244	Trinidad and Tobago	0.361
Hungary	0.152	Tunisia	0.198
Iran	0.547	United Kingdom	0.094
Israel	0.081	United States	0.089
Italy	0.115	Uruguay	0.238
Jamaica	0.041	Venezuela, RB	0.426
Japan	0.075		

Sources: Sturgill (2012) and author's calculations.

Table A5: Human Capital's Share, 2000

Country	Human Capital's Share	Country	Human Capital's Share
Argentina	0.282	Korea, Rep.	0.510
Australia	0.419	Latvia	0.345
Austria	0.416	Mexico	0.264
Belgium	0.420	Moldova	0.335
Bolivia	0.494	Morocco	0.251
Botswana	0.197	Mozambique	0.080
Brazil	0.300	Namibia	0.342
Bulgaria	0.282	Netherlands	0.420
Canada	0.406	New Zealand	0.404
Chile	0.322	Niger	0.128
Colombia	0.310	Norway	0.353
Costa Rica	0.435	Panama	0.289
Cote d'Ivore	0.207	Philippines	0.354
Denmark	0.420	Portugal	0.399
Egypt, Arab Rep.	0.227	Romania	0.490
Estonia	0.369	Russian Fedaration	0.369
Finland	0.346	Spain	0.414
France	0.413	Sri Lanka	0.584
Germany	0.427	Sweden	0.461
Greece	0.324	Switzerland	0.430
Honduras	0.318	Trinidad and Tobago	0.307
Hungary	0.397	Tunisia	0.308
Iran	0.201	United Kingdom	0.415
Israel	0.452	United States	0.501
Italy	0.344	Uruguay	0.348
Jamaica	0.544	Venezuela, RB	0.221
Japan	0.417		

Sources: Sturgill (2012) and author's calculations.

Table A6: Raw Labor's Share, 2000

Country	Raw Labor's Share	Country	Raw Labor's Share
Argentina	0.189	Korea, Rep.	0.260
Australia	0.201	Latvia	0.192
Austria	0.241	Mexico	0.190
Belgium	0.230	Moldova	0.195
Bolivia	0.350	Morocco	0.336
Botswana	0.135	Mozambique	0.324
Brazil	0.244	Namibia	0.355
Bulgaria	0.170	Netherlands	0.221
Canada	0.207	New Zealand	0.201
Chile	0.197	Niger	0.503
Colombia	0.224	Norway	0.185
Costa Rica	0.280	Panama	0.182
Cote d'Ivore	0.302	Philippines	0.207
Denmark	0.254	Portugal	0.302
Egypt, Arab Rep.	0.216	Romania	0.296
Estonia	0.183	Russian Fedaration	0.170
Finland	0.227	Spain	0.250
France	0.240	Sri Lanka	0.337
Germany	0.245	Sweden	0.236
Greece	0.201	Switzerland	0.267
Honduras	0.271	Trinidad and Tobago	0.208
Hungary	0.211	Tunisia	0.300
Iran	0.152	United Kingdom	0.258
Israel	0.236	United States	0.221
Italy	0.213	Uruguay	0.232
Jamaica	0.341	Venezuela, RB	0.196
Japan	0.212		

Sources: Sturgill (2012) and author's calculations.

A4. Why the value of the weight on physical capital per worker matters.

Total capital's share usually serves as the weight on physical capital per worker, and because total capital's share includes the fraction of income accruing to natural capital, the weight assigned to physical capital per worker is artificially high. Therefore, the variance of the physical capital component, $\mathrm{var}[obs_{k,i}]$, is artificially high. When physical capital's share is separated from natural capital's share and inserted in place of total capital's share as the weight on physical capital per worker, the value of $\mathrm{var}[obs_{k,i}]$ falls.

Table A7 provides values for select variances and covariances. Notice that for the constant share scenario, the insertion of physical capital's share for total capital's share reduces $\mathrm{var}[obs_{k,i}]$ from 0.126 to 0.034. If shares are allowed to vary, inserting physical capital's share in place of total capital's share reduces $\mathrm{var}[obs_{k,i}]$ from 0.142 to 0.025. In terms of equation (14), incorporating natural capital yields the additional variance term, $\mathrm{var}[obs_{z,i}]$, and two additional covariance terms, $2\,\mathrm{cov}[obs_{k,i}, obs_{z,i}]$ and either $2\,\mathrm{cov}[obs_{h,i}, obs_{z,i}]$ or $2\,\mathrm{cov}[obs_{h-1,i}, obs_{z,i}]$. Table A7 reveals that all of these covariance terms are positive. Therefore, all new terms in equation (14) have the effect of increasing $\mathrm{var}[observables]$. However, the magnitude of this increase is smaller than the magnitude of the decrease in $\mathrm{var}[obs_{k,i}]$, so the net effect is a decline in $\mathrm{var}[observables]$.[3]

The update to the value of the weight on physical capital per worker drives the change in the variation of observables. The importance of the weight on physical capital per worker is revealed by comparing the variation in each of the factors of production. Recall that $\phi_{n,i} = \ln x_{n,i} - \overline{\ln x_n}$ is the factor portion of $obs_{n,i}$ where x_n represents factor of production n relative to raw labor. In this 53 country sample, the variances of $\phi_{k,i}, \phi_{z,i}, \phi_{h,i}$ and $\phi_{h-1,i}$ are 0.788, 0.346, 0.050 and 0.285, respectively. The cross-country variation in physical capital per worker is much larger than that of the other factors of production. The dispersion of *k* as measured by the variation in $\ln(k_i)$ is over twice as big as the dispersion of *z*, almost sixteen times greater

[3] Incorporating natural capital and adjusting the value of the share associated with physical capital impacts the variation accruing to observables through an additional channel, the correlation coefficient. The intuition follows directly from equation (13). As the magnitude of the correlation between observables and the residual increases (decreases), the fraction of variation in output assumed to reflect variation in observables decreases (increases). Relative to the standard specifications in columns 2 and 3 of Table A7, including natural capital increases the magnitude of the correlation between observables and the residual in all specifications except the one in column 6.

than the dispersion of *h*, and almost three times greater than the dispersion of *h-1*. Because physical capital per worker varies so much, assigning the correct share value as a weight on physical capital per worker is imperative.

Table A7: Select Variances and Covariances

	Factors of Production						
	k and *h*			*k*, *h* and *z*		*k*, *h-1* and *z*	
	Constant Shares: standard assumption of $\alpha_i = \alpha_i + \gamma_i$ = 1/3 and $\beta_i + \eta_i$ = 2/3 for all *i*	Constant Shares: $\alpha_i = \alpha_i + \gamma_i$ = 0.400 and $\beta_i + \eta_i$ = 0.600 for all *i*	Variable Shares: $\alpha_i + \gamma_i$ and $\beta_i + \eta_i$	Constant Shares: α_i = 0.208, γ_i = 0.192 and $\beta_i + \eta_i$ = 0.600 for all *i*	Variable Shares: α_i, γ_i and $\beta_i + \eta_i$	Constant Shares: α_i = 0.208, γ_i = 0.192 and β_i = 0.359 for all *i*	Variable Shares: α_i, γ_i and β_i
Variances and Covariances	**1**	**2**	**3**	**4**	**5**	**6**	**7**
var[*output*]	0.669	0.669	0.669	0.669	0.669	0.669	0.669
var[*observables*]	0.178	0.218	0.234	0.133	0.127	0.174	0.140
var[*residual*]	0.247	0.218	0.228	0.306	0.320	0.275	0.309
var[$obs_{k,i}$]	0.088	0.126	0.142	0.034	0.025	0.034	0.025
var[$obs_{h,i}$]	0.022	0.018	0.017	0.018	0.017		
var[$obs_{(h-1),i}$]						0.037	0.022
var[$obs_{z,i}$]				0.013	0.019	0.013	0.019
cov[$obs_{k,i}$, $obs_{h,i}$]	0.034	0.037	0.038	0.019	0.015		
cov[$obs_{k,i}$, $obs_{(h-1),i}$]						0.023	0.018
cov[$obs_{k,i}$, $obs_{z,i}$]				0.009	0.011	0.009	0.011
cov[$obs_{h,i}$, $obs_{z,i}$]				0.005	0.007		
cov[$obs_{(h-1),i}$, $obs_{z,i}$]						0.013	0.009
Raw Correlation							
correlation coefficient, $\rho_{obs_{\cdot i}, res_{\cdot i}}$	0.580	0.534	0.447	0.571	0.553	0.502	0.532

Observables refers to the translog multilateral input per worker index. *Residual* is the translog multilateral productivity index. *Output* is the translog multilateral output per worker index.

$\rho_{obs_{\cdot i}, res_{\cdot i}}$ is the raw correlation between *observables* and the *residual*.

$obs_{n,i} = \frac{1}{2}\left(s_{n,i} + \overline{s_n}\right)\left(\ln x_{n,i} - \overline{\ln x_n}\right)$ where x_n represents factor of production *n* relative to raw labor, s_n is the corresponding factor share, and a bar above a variable indicates the mean of that variable across all countries in the sample.

k, *h*, *h-1* and *z* denote physical capital per worker, effective labor per worker, human capital per worker and natural capital per worker, respectively.

$\alpha + \gamma$ is *total* capital's share. α is physical capital's share, and γ *is* natural capital's share. $\beta + \eta$ is *total* labor's share. β is human capital's share, and η is raw labor's share.

The mean values of α, γ, β and η are 0.208, 0.192, 0.359 and 0.241, respectively.

A5. The zero lower bound on the range of variation in output accruing to factors shares

Notice that the lower bound for the range of variation in output accruing to each factor share is always zero. This should not be viewed as evidence of factor share variation being unimportant. The lower bound equals zero by construction and so it equals zero irrespective of the size of the factor share variance and the strength of the correlation between factor shares and output per worker. The reliance of the translog multilateral index on differences, the decomposition of the variance of a product of dependent variables, and the lack of a theory to guide the allocation of the interaction between factor shares and factors creates the zero lower bound. Recall that if all interaction between $\psi_{n,i}$ and $\phi_{n,i}$ is assumed to reflect variability in $\phi_{n,i}$, the relative variance decomposition for the observable component $(obs_{n,i})$ is given by equation (A7). The first term on the left hand side of equation (A7) is the fraction of variation in $obs_{n,i}$ attributable to variation in $\psi_{n,i}$, the share portion of $obs_{n,i}$. The numerator of this term is a product that contains $E^2(\phi_{n,i})$, which is always equal to zero.

The theory of factor saving innovations explored by Zuleta (2008) and Peretto and Seater (2013) says that factors and factor shares are correlated. The causality runs in both directions. There is a feedback effect, and factors and factor shares drive each other. Because of this, neither of the extreme allocations considered in section A2 is correct. However, there is nothing that suggests how the covariance between factors and factor shares should be allocated. Acknowledging that the explanatory power of each factor and factor share falls somewhere within the upper and lower bounds but never equals either bound is the most accurate determination.

A6. Development accounting results for $Ratio_1$

Table A8: Development Accounting Results

***Residual* assumed constant across countries**

	Factors of Production						
	k and *h*			*k*, *h* and *z*		*k*, *h-1* and *z*	
	Constant Shares: standard assumption of $\alpha_i = \alpha_i + \gamma_i$ = 1/3 and $\beta_i + \eta_i$ = 2/3 for all *i*	Constant Shares: $\alpha_i = \alpha_i + \gamma_i$ = 0.400 and $\beta_i + \eta_i$ = 0.600 for all *i*	Variable Shares: $\alpha_i + \gamma_i$ and $\beta_i + \eta_i$	Constant Shares: α_i = 0.208, γ_i = 0.192 and $\beta_i + \eta_i$ = 0.600 for all *i*	Variable Shares: α_i, γ_i and $\beta_i + \eta_i$	Constant Shares: α_i = 0.208, γ_i = 0.192 and β_i = 0.359 for all *i*	Variable Shares: α_i, γ_i and β_i
Variance Decomposition	**1**	**2**	**3**	**4**	**5**	**6**	**7**
$Ratio_1$	0.267	0.326	0.350	0.199	0.190	0.260	0.208
Variation accruing to $\ln(k_i)$	0.052-0.253	0.075-0.315	0.077-0.339	0.010-0.157	0.007-0.135	0.009-0.200	0.007-0.146
Variation accruing to $\ln(h_i)$	0.013-0.214	0.011-0.251	0.010-0.260	0.008-0.133	0.007-0.117		
Variation accruing to $\ln(h_i-1)$						0.013-0.196	0.007-0.137
Variation accruing to $\ln(z_i)$				0.013-0.077	0.011-0.094	0.012-0.090	0.010-0.101
Variation accruing to $\alpha_i + \gamma_i$			0.000-0.047				
Variation accruing to $\beta_i + \eta_i$			0.000-0.013		0.000-0.006		
Variation accruing to α_i					0.000-0.019		0.000-0.020
Variation accruing to β_i							0.000-0.030
Variation accruing to γ_i					0.000-0.035		0.000-0.037

$Ratio_1$ = (variation in *observables*)/(variation in *output*). All countries are assumed to have the same *residual* value, and the correlation between *observables* and the *residual* is ignored.

Observables refers to the translog multilateral input per worker index. *Residual* refers to the translog multilateral productivity index. *Output* refers to the translog multilateral output per worker index.
k, *h*, *h-1* and *z* denote physical capital per worker, effective labor per worker, human capital per worker and natural capital per worker, respectively.

$\alpha + \gamma$ is total capital's share. α is physical capital's share, and γ *is* natural capital's share. $\beta+\eta$ is *total* labor's share. β is human capital's share, and η is raw labor's share.

The mean values of α, γ, β and η are 0.208, 0.192, 0.359 and 0.241, respectively.

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