back then: Here I am, a dynamicist, teaching a course in my specialty, differential equations, but conveying NONE of the excitement that was swirling around dynamics in those days.

My current course is much different. In fact, I think that the evolution in MY differential equations course has already happened. Yes, it will evolve somewhat over the next few years. Yes, technological breakthroughs will influence what and how I teach. But I doubt that the curricular changes that will occur over the next decade will come anywhere close to the changes that have already occurred.

Here is an abridged version of my syllabus from 1982 including the number of weeks (approximately) I spent on each topic:

- First order equations (Separable, exact, linear, Bernoulli, applications) --- 2 weeks
- Second order equations --- 4 weeks
- Higher order equations --- 1 week
- Laplace transforms --- 2 weeks
- Power series solutions --- 2 weeks
- Systems --- 2 weeks.

If I recall, I really wanted to spend the 2 weeks on this last topic, but, as always happens, I had to cut a lot of it due to lack of time.

Comparing this syllabus to the syllabus I used in 1994, the only topics that remained were:

- First order equations (Separable, linear, applications)
- Systems

Yes, power series, Laplace transforms, second order equations were all gone from the syllabus! Obviously, this leaves a lot of room for change.

In 1994 my syllabus had become:

- First order equations (separable, linear, applications, as before). Now also slope fields, numerical methods, existence/uniqueness, phase lines, solution graphs, bifurcations --- 3 weeks
- Systems. Right to systems; no stop for second order equations. Predator-prey, nullclines, phase plane, numerical methods, linear systems, second order equations (including forcing, resonance), applications, bifurcations of linear systems --- 7 weeks
- Nonlinear equations. Linearization, complete qualitative analysis, Lorenz equations --- 2 weeks
- Discrete dynamics. Iteration, web diagrams, chaos --- 1 week.

This is only a rough outline of the syllabus, but note how different it is from my 1982 syllabus.

Obviously, this syllabus will not work for everyone, but there is plenty of room for modification (Laplace transforms in place of discrete dynamics or less on nonlinear systems). The point is that this is a VERY different course from the one I taught 10 years ago.

How will this syllabus evolve in the future? Technology will dictate some changes, but I suspect that the basic outline will remain the same. I hope to add a little more on discrete dynamics as soon as our calculus course opens up some room and covers more of the material included in first order ODEs. Then I hope to make a chaotic nonlinear system such as the Lorenz equations or the Chua circuit the capstone of the course. I wonder if it is possible to introduce one of these two systems, compute the local behavior near equilibrium points, observe a Poincare section, and then reduce to a one-dimensional mapping? With enought discrete dynamics, the chaotic behavior of these systems could then be effectively analyzed rather than simply viewed in gee-whiz fashion. If possible, that would be great! With a little more in the realm of discrete dynamics, I could then give my students a real taste of what is currently of interest in mathematics, and, at the same time, introduce them to the modern theory of differential equations. ❑

DETECTING A LEAK IN AN UNDERGROUND STORAGE TANK

E. Aboufadel and S.J. Tavener

An *inverse* problem is one in which the parameters of a model are to be determined from data obtained by experiment. This is the opposite of the more usual *forward* problem which seeks to calculate the outcome of a model given the parameters of the model. The most familiar example of an inverse problem is least-squares linear regression, in which the slope and y-intercept of an assumed linear model are to be determined from a number of data pairs. More complex inverse problems are found in diverse areas, from geophysics to medicine. Usually, there are more data points than parameters of the model to be determined, as in the case of least-squares regression. The converse may also be true. When using data from a multiple breath nitrogen washout experiment to fit a multi-compartment lung model (the patient is given a mixture of oxygen and an inert gas to breathe and the concentration of nitrogen in each subsequent exhalation is monitored), there are typically fewer data points than parameters to be determined. One approach for solving this problem is discussed in Ryan and Tavener (1985) [1].

We developed the following inverse problem while attending the NSF-sponsored conference "Teaching ODEs with Computer Experiments" held at RPI in July 1994. This laboratory, suitable for a sophmore-level course, presents the student with an interesting approach to the traditional mixing problem.

Problem definition

A tank is filled with salty water and is believed to be leaking, potentially polluting nearby groundwater. It is not possible to measure the volume of fluid in the tank directly, but it is possible to take measurements of the concentration of the salt solution in the tank at various times. In order to determine whether or not the tank is leaking, the tank is flushed with a solution containing 0.05 pounds of salt per gallon, at a rate of one gallon per minute, beginning at time $t = 0$. Fluid is removed from the tank at the same rate. The salt concentration of the outflow is measured at $t = 0$, and after 10 and 20 minutes. The salt concentration in the tank is initially 0.2 pounds per gallon, but drops to 0.18 and 0.16 pounds per gallon after 10 and 20 minutes respectively. Assuming complete and instantaneous mixing of the inflow and the fluid resident in the tank, and assuming that the leak (if present) is constant, determine the rate at which the tank is leaking.

Solution

The solution of this problem, while conceptually simple, involves both the solution of a non-trivial differential equation and the solution of a pair of simultaneous, nonlinear equations. For these reasons, MAPLE (or some other computer algebra system) is an appropriate tool. We present an outline of a solution of this problem.

Let us first define the following vatiables:

- R_{in} = the rate at which fluid is added to the tank (in gallons per minute)
- R_{out} = the rate at which fluid is withdrawn from the tank (in gallons per minute)
- R_{leak} = the rate at which fluid is leaking from the tank (in gallons per minute)
- C_{in} = the salt concentration of the fluid being added to the tank (in pounds per gallon)
- C_0 = the salt concentration of the fluid at time $t = 0$ (in pounds per gallon)
- $V(t)$ = the volume of liquid in the tank at time t (in gallons)
- $Q(t)$ = the amount of salt in the tank at time t (in pounds)
- $C(t)$ = the salt concentration in the tank at time t (in pounds per gallon)

The amount of salt in the system is conserved, hence the rate of change of the amount of salt present is the difference between the rate at which salt is added and the rate at which salt is removed, or

$$\frac{dQ}{dt} = R_{in}C_{in} - C(t)(R_{out} + R_{leak}). \tag{1}$$

The salt concentration $C(t)$ may be expressed in terms of the amount of salt present and the volume as

$$C(t) = \frac{Q(t)}{V(t)}. \tag{2}$$

The volume $V(t)$ depends upon the initial volume V_0 and the rates of inflow, outflow and leakage, as

$$V(t) = V_0 + (R_{in} - R_{out} - R_{leak})t. \tag{3}$$

Finally, the initial volume may be written in terms of the amount of salt present at $t = 0$ and the initial salt concentration, as

$$V_0 = \frac{Q_0}{C_0}. \tag{4}$$

Combining these equations gives

$$\frac{dQ}{dt} = R_{in}C_{in} - \frac{Q(t)(R_{out} + R_{leak})}{\frac{Q_0}{C_0} + (R_{in} - R_{out} - R_{leak})t}. \tag{5}$$

Solving the differential equation (5) for $Q(t)$, and substituting the known values of R_{in}, R_{out}, C_{in} and C_o, we have

$$Q = 0.25Q_0\left(1 + 3\left(\frac{5Q_0 - tR_{leak}}{5Q_0}\right)^{\alpha}\right) - 0.05tR_{leak}. \tag{6}$$

$$\text{where } \alpha = \frac{1 + R_{leak}}{R_{leak}}.$$

The salt concentration $C(t)$ may now be computed from equation (2) using (3),(4), and (6), and the known values of R_{in}, R_{out}, C_{in} and C_0. We have

$$C(t) = 0.05\left\{1 + 15\beta^{\alpha - 1} Q_0^{\left(\frac{-1}{R_{leak}}\right)} e^{(-1.61\alpha)}\right\} \qquad (7)$$

where $\beta = -5Q_0 + tR_{leak}$

Expressions for the concentrations at $t = 10$ and $t = 20$ in terms of the unknown values of Q_0 and R_{leak} may be developed by evaluating (7) at $t = 10$ and $t = 20$. Using the measured salt concentrations at $t = 10$ and at $t = 20$ then yields two simultaneous nonlinear equations for Q_0 and R_{leak} . The solution of this system is

$$R_{leak} = 1.0 \text{ and } Q_0 = 15.0 .$$

We have plotted the salt concentrations as a function of time for the computed leakage rate and for the no leak scenario in the figure below. The solid line shows the concentration in the leaking tank as a function of time, with the two boxes indicating the two given data points. The dashed line shows the decay of the salt concentration under the same circumstances when the tank is free of leaks. The horizontal dashed curve shows the concentration of salt solution used to flush the tank.

Figure 1 Salt concentration in the tank

An interesting extension to this problem is the question of how best to incorporate more data. For instance, if the salt concentration is also measured at $t = 30$ minutes, how should this extra information be used? An energetic student may wish to consider the sensitivity of the solution to the given data. ❑

Editor's note: Another good exercise, at the very beginning, is to ask the students to provide some "real life" scenario that might fit the model.

[1] Ryan, D.M. Tavener, S.J. 1985 "Bounds on smooth solutions of underdetermined linear models for gas exchange," *Journal of Optimization and its Applications* **47,** 349--368

Small Mammal Dispersion

Ray Huffaker, Thomas LoFaro, and Kevin Cooper

Setting the Scene

Beavers, hunted in open access for their pelts, were saved from extinction in the middle of this century by regulations controlling trapping season, method and numbers. Under this protection, the beaver population has rebounded in many regions of the country and has caused significant damage to valuable timber and agricultural land. Trapping is most effective in controlling beavers whose primary nuisance is tree-cutting on privately-held timber land.

A trapping strategy that disregards the possible migratory behavior of beavers in neighboring "uncontrolled" (i.e., zero trapping) land parcels in filling the vacuum created by trapping in the "controlled" parcel, can be as futile in practice as attempting to dig a hole in fine-grain sand. We formulate a two-equation system of differential equations to model this phenomenon according to the recently formulated "social-fence" hypothesis of small mammal dispersion. This hypothesis can be viewed as the ecological analog of osmosis: Beavers from an environmentally superior habitat are posited to diffuse through a social fence to an inferior but less-densely populated habitat until the pressure to depart ("within-group aggression") is equalized with the pressure exerted against invasion ("between-group aggression"). This is termed "forward migration." Assuming that the controlled parcel is a superior habitat, the owner must be concerned with the "backward migration" that occurs when the superior parcel becomes less densely populated through trapping.

Rate Equations

Let X and Y represent the population densities [head (hd)/square mile (sq mi)] of beavers in the controlled and uncontrolled parcels, respectively; and let $\dot{X}$ and $\dot{Y}$ represent the associated annual net rates of change [hd/sq mi/year (yr)]. The following pair of differential equations models $\dot{X}$ and $\dot{Y}$ as the difference between the rates of net growth (i.e., birth rate minus the death rate), dispersion, and, in the case of X, trapping:

$$\dot{X} = F_0(X)X - F_1(X, Y) - PX \qquad (1)$$