
A New Method for Computing the Mean Center of Population of the United States

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The mean center of population of the United States is a convenient location used to summarize the population distribution of the United States and how it changes over time. As computed by the U.S. Census Bureau, the center depends on an arbitrary choice of a map projection. We feel that this location should depend only on the population distribution and not on any choices made in representing the data of the distribution. This note discusses a method for computing this location that does not depend on any choices made and describes how the results of this method differ from those of the Census Bureau. **Key Words:** center of population, U.S. census, U.S. Census Bureau.

In its decennial reports, the U.S. Census Bureau computes the mean center of population of the United States, a location designed to summarize the nation's population distribution and its change over time. This location usually receives considerable attention in the media (De Jonge 2001; *National Geographic* 2002). The method used by the Bureau to compute this location first projects points from the three-dimensional sphere to a flat map using a sinusoidal projection and then averages the two-dimensional coordinates. Of course the choice of a different map projection leads to a different location for the center of population. We, therefore, propose using a method that does not depend on any choice of map projection. In this note, we explain our method and compare it to the Census Bureau's method. We will see that the center computed by the Census Bureau in the 2000 Census lies roughly 125 km south of the center computed with our method. This discrepancy can be expected to increase, with the Census Bureau's center drifting further south as the population becomes more spread out across the country.

The general problem of defining a center of population has been discussed by others, most notably by Hayford (1902) and Bachi (1999). The particular problems arising from the Census Bureau's computation have been noted previously by Barmore (1993), who suggested another method for computing the center of population. Indeed, the bibliography of Bar-

more's paper provides a good introduction to this question. We describe and examine Barmore's definition later in this article.

The Census Bureau has described its aim in computing this center of population point:

The concept of the center of population as used by the U.S. Census Bureau is that of a balance point. That is, the center of population is the point at which an imaginary, weightless, rigid, and flat (no elevation effects) surface representation of the 48 conterminous states and the District of Columbia (or 50 states as appropriate to the computation) would balance if weights of identical size were placed on it so that each weight represented the location on [sic] one person.

(U.S. Census Bureau 2001)

Defining the center of population as a balance point is reasonable since the resulting location has a simple intuitive meaning and may be easily computed. Because the data points lie on Earth's curved surface, however, the meaning of the word "flat" requires some interpretation. As we will explain, the most unbiased interpretation is one that does not rely on any choices made in representing the data of the population distribution.

Beginning in 1960, the Census Bureau has computed the center of population by dividing the country into many small geographic areas (eight million in the 2000 Census). For each of these areas, the population is denoted by w_i , the latitude by ϕ_i , and the longitude by λ_i . If the center of population is the point with latitude $\bar{\phi}$

and longitude $\bar{\lambda}$, then the Bureau computes

$$\bar{\phi} = \frac{\sum_i w_i \phi_i}{\sum_i w_i} \quad \bar{\lambda} = \frac{\sum_i w_i \lambda_i \cos \phi_i}{\sum_i w_i \cos \phi_i}. \quad (1)$$

As Barmore (1993) has noted, this computation produces the point at which a flat map would balance if the population of the United States were placed on the map using a sinusoidal (Sanson-Flamsteed) projection whose central meridian is placed at longitude $\bar{\lambda}$. (Snyder [1987] has described many map projections, including those mentioned in this article.) One notices that the latitude is simply averaged, whereas the longitude is computed as an average in which each person is weighted by the cosine of his or her latitude, $\cos(\phi_i)$. However, because the range of latitudes spanned by the United States is not large, this is not a significant factor: the difference between $\bar{\lambda}$ and the average longitude, computed simply as $\sum_i w_i \lambda_i / \sum_i w_i$, is approximately six minutes of longitude for the 2000 Census, a difference of only 8 km.

There appears to be no compelling reason for choosing the sinusoidal projection over another projection. Indeed, choosing different projections to represent the data leads to different locations for the center of population. Of course a well-known consequence of Gauss's (1828) *Thereoma Egregium* is that it is not possible to represent Earth's surface on a flat map without distorting distances (see Gauss's theorem and a discussion of its significance in O'Neill 1966). Because a balance point depends on distances, any computation that first projects the data onto a flat map will not give the actual balance point. For this reason, we propose considering this computation from a three-dimensional perspective and finding the balance point of the population distribution as it resides on an idealized spherical representation of Earth's surface. As we will see, the result is a location that is easily computed and does not depend on a choice of map projection.

Following the Census Bureau, we divide the country into regions whose population is w_i and whose location is represented by a three-dimensional vector $\mathbf{x}_i$ extending from Earth's center to the center of the region on Earth's surface. For convenience, we choose one unit of distance to equal Earth's radius and we follow the Census Bureau in assuming that Earth is a perfect sphere. To compute a balancing point, we recall that the gravitational force on the population of

this region can be denoted as $\mathbf{F}_i = -k w_i \mathbf{x}_i$, where k is a constant that depends on the mass and radius of the Earth. The total force on the population is then $\mathbf{F} = \sum_i \mathbf{F}_i = -k \sum_i w_i \mathbf{x}_i$, and the balance point will be the location at which this total force is directed toward Earth's center. This occurs at the location $\bar{\mathbf{x}}$ where the total force $\mathbf{F}$ is parallel to $\bar{\mathbf{x}}$, and consequently $\mathbf{F} = -|\mathbf{F}| \bar{\mathbf{x}}$, where $|\mathbf{F}|$ is the length of $\mathbf{F}$. This means that

$$\bar{\mathbf{x}} = \frac{-\mathbf{F}}{|\mathbf{F}|} = \frac{\sum_i w_i \mathbf{x}_i}{|\sum_i w_i \mathbf{x}_i|}. \quad (2)$$

To compute this location, we first set up a three-dimensional coordinate system on Earth. The origin of this coordinate system is at Earth's center, the positive x -axis runs through the intersection of the prime meridian and the equator, the positive y -axis runs through the intersection of the longitude at 90° East and the equator, and the positive z -axis passes through the North Pole. One unit of distance is equal to Earth's radius. In this way, a longitude-latitude pair (λ_i, ϕ_i) gives the three-dimensional vector $\mathbf{x}_i = (x_i, y_i, z_i)$ through the relations

$$\begin{aligned} x_i &= \cos \lambda_i \cos \phi_i \\ y_i &= \sin \lambda_i \cos \phi_i \\ z_i &= \sin \phi_i \end{aligned}$$

To find the balance point, we then compute the balance point $\bar{\mathbf{x}} = \frac{\sum_i w_i \mathbf{x}_i}{|\sum_i w_i \mathbf{x}_i|}$ as the vector whose coordinates are

$$\bar{x} = \frac{\sum_i w_i x_i}{|\sum_i w_i \mathbf{x}_i|} \quad \bar{y} = \frac{\sum_i w_i y_i}{|\sum_i w_i \mathbf{x}_i|} \quad \bar{z} = \frac{\sum_i w_i z_i}{|\sum_i w_i \mathbf{x}_i|}.$$

From here, we can recover the latitude and longitude of the center of population using

$$\begin{aligned} \hat{\phi} &= \sin^{-1} \bar{z} \\ \hat{\lambda} &= \tan^{-1} \frac{\bar{y}}{\bar{x}} \end{aligned}$$

We note that the point given by Equation (2) is also given by finding the weighted average of the coordinates; that is,

$$\hat{x} = \frac{\sum_i w_i x_i}{\sum_i w_i}, \quad \hat{y} = \frac{\sum_i w_i y_i}{\sum_i w_i}, \quad \hat{z} = \frac{\sum_i w_i z_i}{\sum_i w_i}.$$

From here, the balancing point is

$$\bar{\mathbf{x}} = \frac{(\hat{x}, \hat{y}, \hat{z})}{|(\hat{x}, \hat{y}, \hat{z})|}.$$

Barmore (1993) discussed this approach to defining a center of population but rejected it because the point $(\hat{x}, \hat{y}, \hat{z})$ usually lies inside the Earth (though $\bar{x}$ is a point on Earth's surface). We feel that the interpretation of $\bar{x}$ as a balancing point, together with the ease with which it may be computed, make it the most appealing candidate for the center of population.

To compare our method with that used by the Census Bureau, let us first consider an unrealistic but simple test in which all of the population of the United States is concentrated in equal numbers in Los Angeles (34N03, 118W15) and New York (40N43, 74W0). The formulas used by the Census Bureau (1) give

$$\bar{\phi} = 37N23, \quad \bar{\lambda} = 97W07,$$

and those from the three-dimensional formula (2) give

$$\hat{\phi} = 37N31, \quad \hat{\lambda} = 97W10.$$

The difference is illustrated in Figure 1. The two points computed are shown, as well as arcs of great circles connecting them to New York and Los Angeles. Notice that if we were to stand at the point computed using the three-dimensional formula with a very light rod connecting us to Los Angeles and another connecting us to New York, the rods would be of equal length and point in opposite directions. This is the condition required for balancing. However, if we were to stand at the point given by the formulas used by the Census Bureau (1), the two rods would not point in opposite directions and therefore would not balance.

It is important to note that the formulas used by the Census Bureau pull the center of population south of its true location. These formulas put the center of population roughly at the midpoint of a segment drawn between the two cities in the longitude-latitude coordinate plane. However, great circles, the paths of shortest distance and therefore the true "straight lines" on a sphere, travel a more northerly route in the Northern hemisphere.

Turning now to the figures for the 2000 Census (see U.S. Census Bureau 2002), we find the location given by the Census Bureau

$$\bar{\phi} = 38N47, \quad \bar{\lambda} = 91W34$$

compared to our computation of

$$\hat{\phi} = 38N42, \quad \hat{\lambda} = 91W49.$$

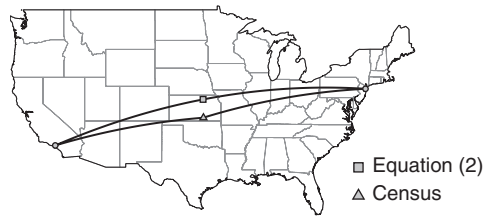


Figure 1 A simple test case.

As shown in Figure 2, the two points differ by some 125 km, with the point computed by the Census Bureau, as expected, lying south of the center given by Equation (2).

When viewed over the history of the Census, the center of population gives an indication of how the distribution of the population is changing. For instance, we certainly expect to see this center move to the west as the population does. For this reason, it is interesting to compute the centers of population from 1790 through 2000 using Equation (2) and compare it to the center reported by the Census Bureau. (Data for this computation are given in U.S. Census Bureau 1993.) The results are shown in Figure 3. As expected, both sets of points move to the west. However, the point computed by the Census veers south more quickly than the center computed using (2). Furthermore, the difference between the two sets of points grows over time. This is because the Census Bureau's formulas result from considering the population as if it

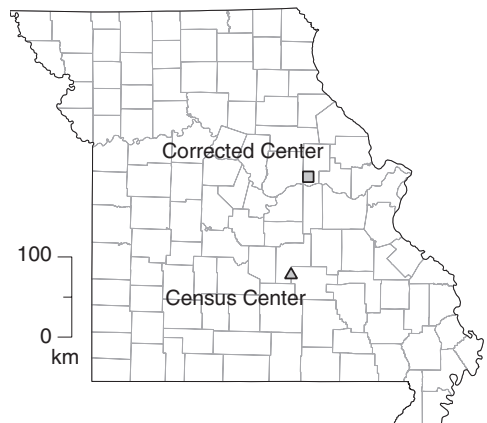


Figure 2 Centers of population for the 2000 Census shown in the Mercator projection.

Typos that were not fixed in the final version. It should read:
The Census Bureau reports 37N42, 91W49 and our
computation gives 38N47, 91W34.

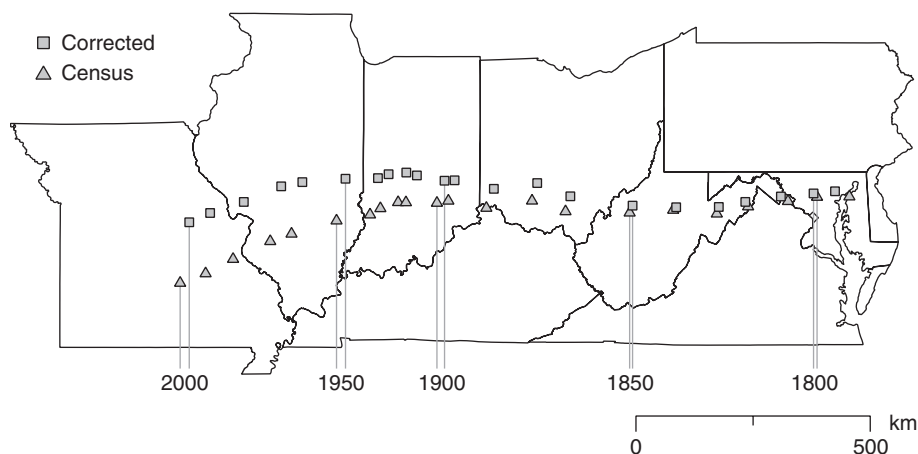


Figure 3 Centers of population, 1790–2000, shown in the Mercator projection.

were laid out in a flat plane. When the population was concentrated in a small area, this was approximately true. However, as the population has become more uniformly distributed across the country, the effects of Earth's curvature have become more pronounced and the southerly drift continues to grow.

The problem with the Census Bureau's formulas was noted by Barmore (1993), who proposed another method for finding the center of population. Barmore's center of population is a point that, when used as the center of an azimuthal equidistant projection, is a balance point for the two-dimensional distribution. Barmore chose this projection because such a map accurately represents the distance between the center and any other point. However, Barmore's definition suffers from a few drawbacks that make it less desirable. First, it is unclear what the intuitive meaning of this point is. A simple example shows that it is different from the balance point. For instance, if we consider an idealized example in which all the population is concentrated in New York and Los Angeles with New York having twice the population of Los Angeles, Barmore's center is located 12 miles west of the balance point as found in (2) above. Second, Barmore's center must be computed through an iterative approach that is technically cumbersome. Finally, it is relatively straightforward to find examples in which this center is not unique.

In summary, the mean center of population is meant as a convenient summary of the popula-

tion distribution. However, the method used by the Census Bureau requires that an arbitrary choice of map projection be used, and this choice results in the center drifting south of a less-biased computation. The method proposed in this paper possesses some advantages in that it has intuitive meaning as a balance point, is free of any choices made in representing the data, and is relatively simple to compute. ■

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