A MODIFIED HANNING WAVELET

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Abstract

The wavelet theory is a powerful method for processing different signals. The Hanning wavelet can be obtained from the Hanning window. In this brief report we present a new wavelet transform which is a generalisation of Hanning window. This transform is a more flexible one because dependent on three parameters and can cover more combinations of discrete and continuous parameter discretisations (hybrid cases).

In processing signals, a part the well known Fourier methods, a more powerful instrument is offered by the wavelet theory[1,2].The Hanning window[3] is a product of two Heaviside functions.Each Heaviside function is given[4] by :

$$ApproxH_a(t) = \frac{1}{2} + \frac{1}{\pi} arctg\left(t/a\right) \tag{1}$$

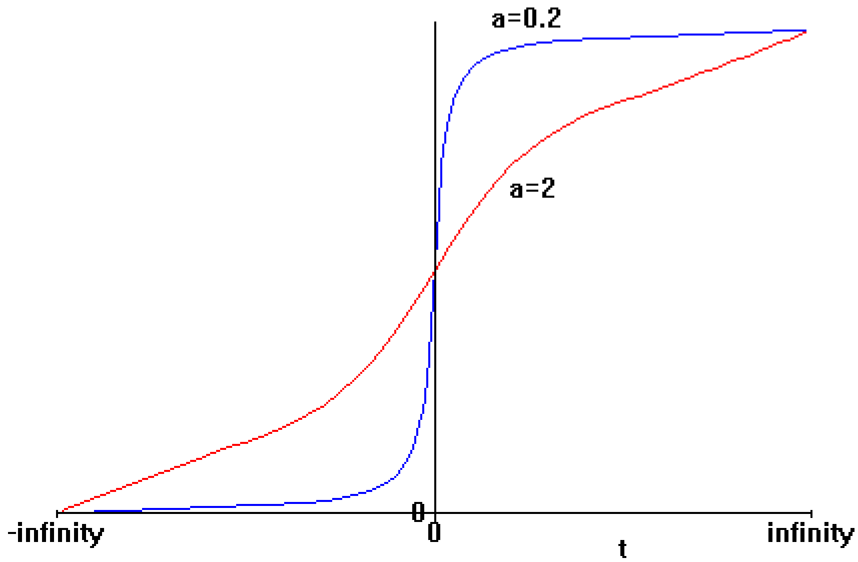
where a is the scale. In figure 1 the Heaviside function is represented for different values of a $\left(a = 0.2\ ; a = 2\right)$ .We construct the mother wavelet[5] which is the second derivative of expression (1) :

$$h_{d,a}(t) = \frac{d^2}{dt^2}\left\{ ApproxH_a(t+d) * ApproxH_a(d-t) \right\} = -2\frac{1}{\pi^2 a^2\left(1+\left(\frac{t+d}{a}\right)^2\right)\left(1+\left(\frac{t+d}{a}\right)^2\right)^2} -$$

$$-2\frac{\left(\frac{1}{2}+\frac{arctg\left(\frac{d-t}{a}\right)}{\pi}\right)(t+d)}{\pi a^3\left(1+\left(\frac{t+d}{a}\right)^2\right)^2} - 2\frac{\left(\frac{1}{2}+\frac{arctg\left(\frac{t+d}{a}\right)}{\pi}\right)(d-t)}{\pi a^3\left(1+\left(\frac{d-t}{a}\right)^2\right)^2} \tag{2}$$

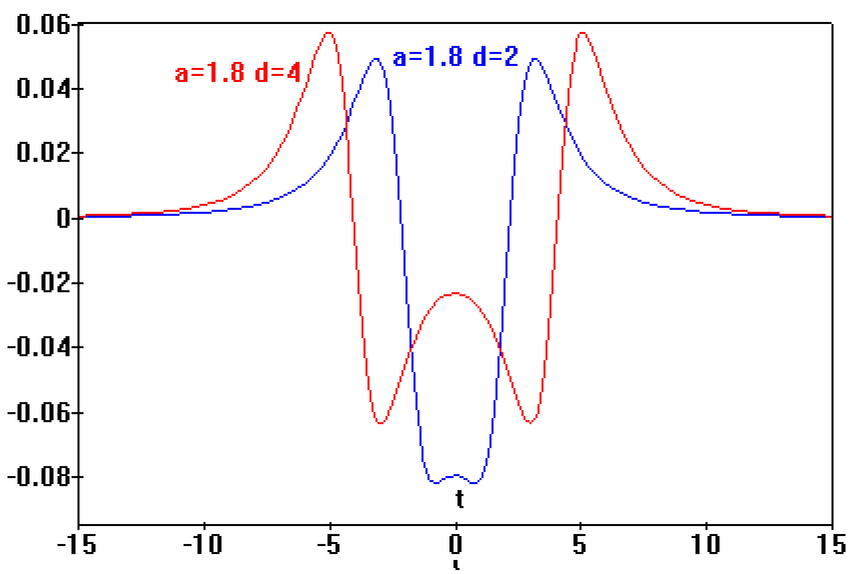
where d is the window length.For the mother wavelet we have: $\int_{-\infty}^{\infty} h_{d,a}(t)dt = 0$ (3)

Fig. 1 Approximation of the Heaviside function

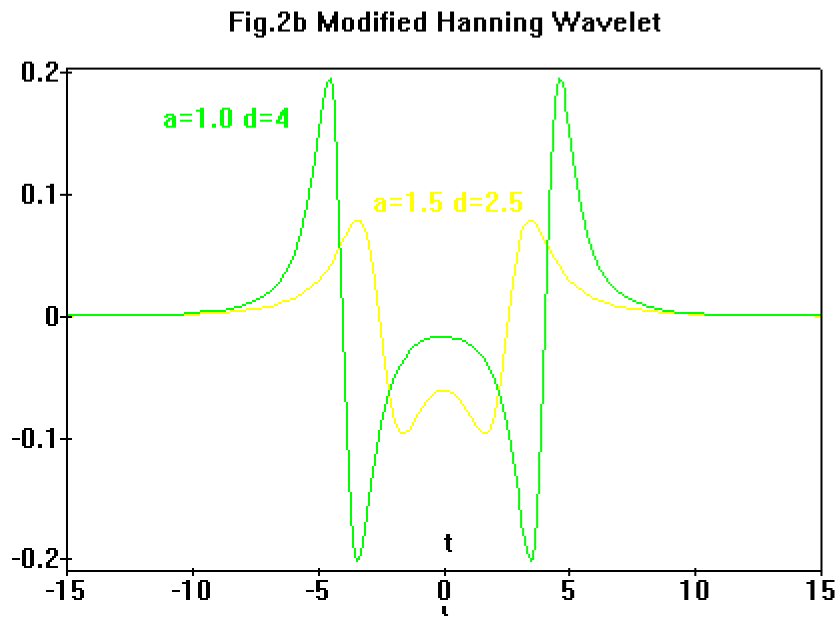


In fig. 2a the mother wavelets $\left(a = 1.8, d = 2; a = 1.8, d = 4\right)$ were shown.

Fig.2a Modified Hanning Wavelet



In fig. 2b the mother wavelets $\left(a = 1.0, d = 4; a = 1.5, d = 2.5\right)$ were shown.



The set of daughter wavelets are generated from the mother wavelet by shift operations :

$h_{a,b,d}(t) = \frac{1}{\sqrt{F}} h_{d,a}(t-b)$ (4a) where b is the shift.The normalisation factor is given by :

$F = \int_{-\infty}^{\infty} \left| h_{d,a}(t) \right|^2 dt$ (4b). The (1-D) wavelet transform of the f(t) is defined as :

$$W(a,b,d) = \int_{-\infty}^{\infty} f(t) h^*_{a,b,d}(t) dt \qquad (5)$$

This is a correlation operation between the signal f(t) and the shifted and scaled mother wavelet $h_{a,b,d}(t)$. In the limit case $d \to 0$ we have the usual wavelet (Mexican hat) which depends of two parameters. In the limit case $b \to 0$ we have the Hanning wavelet. The $h_{a,b,d}(t)$ wavelet depends of three parameters (a,b,d). Also this wavelet generalize Mexican and Hanninng wavelets and is more flexible then this ones.

Acknowledgements

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