Homework: Accelerating reference frames: Inertial "forces" and local acceleration due to gravity

Name ________________________

1. Imagine that you are flying to Maui for vacation during winter break (lucky you). In the situations described in parts a and b below:

   - draw a sketch clearly indicating the relative directions of the actual gravitational force and the inertial "force" you feel (as measured in the frame of the airplane),
   - clearly indicate the (approximate) direction of the local acceleration due to gravity (as measured in the frame of the plane), and
   - determine whether the magnitude of your local acceleration due to gravity would be *greater than, less than,* or *equal to* the actual acceleration due to gravity.

   In each case, explain how you determined your answers.

   a. Your plane turns by banking to the left, flying at constant altitude and constant speed. (*Hint:* Start by sketching a "front view" or "rear view" diagram of the plane.)

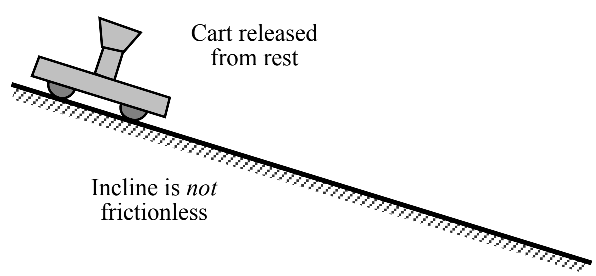
   b. Your plane slows down as it follows a straight-line, descending trajectory as it lands in Maui. (*Hint:* Start by sketching a "side view" diagram of the plane.)

2. Recall the situation from section II of the tutorial: A cart is fitted with a mechanism that launches a ball from the top of the cart. If the cart remains at rest on a horizontal surface while the ball is launched, the ball drops back down into the launcher.

   Suppose that the cart were released from rest on a long incline before the ball is launched. However, assume that the incline is *not frictionless.* (Ignore any effect due to the rotation of the wheels of the cart.)

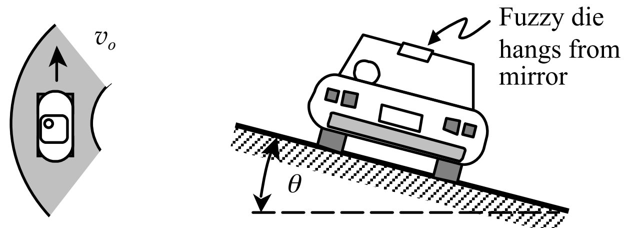
   a. As observed in the frame of the cart, how does the direction of the local acceleration due to gravity compare to the orientation of the incline? Support your answer with one or more free-body diagrams drawn in the frame of the cart.

   b. Does the ball drop *back into* the launcher, *to the left of* it, or *to the right of* it? Carefully explain how your work in part a can be used to justify your answer.



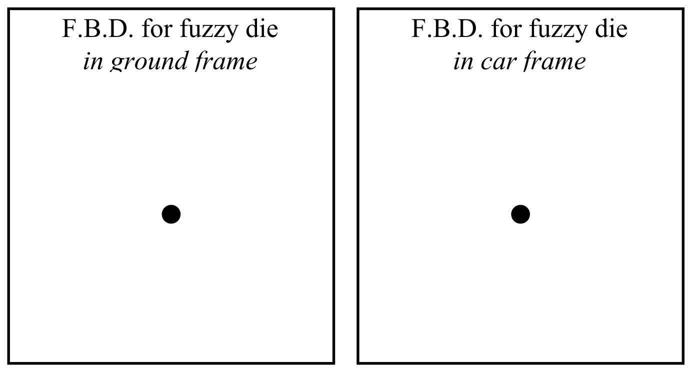
3. A car travels along a circular track of radius D that is banked at an angle θ with respect to horizontal. The car moves with constant speed v_o relative to the track. A single fuzzy die (*not* shown in the diagrams below) is suspended from a string tied to the rear view mirror of the car. (Originally there were two fuzzy dice, but the string for the other die broke some time ago.).

   Note: In answering all parts of this problem, ignore the rotation of the Earth.



Top view Rear view

   a. Draw free-body diagrams for the fuzzy die (a) in the frame of the *ground* and (b) in the frame of the *car*. Draw your diagrams so that they correspond to the instant shown in the pictures above. Clearly label all forces, including fictitious "forces."

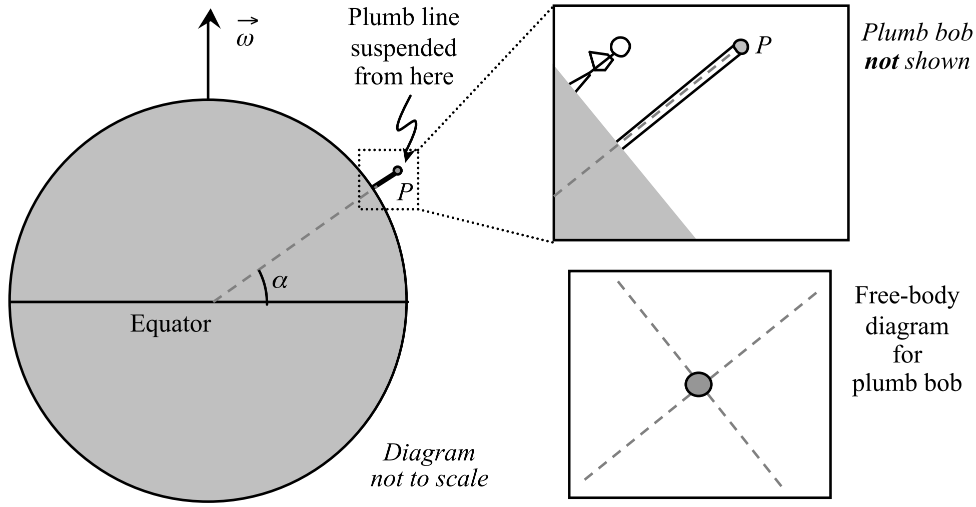


   b. In the frame of the <u>car</u>, is the magnitude of the local acceleration due to gravity *greater than, less than,* or *equal to* that of the actual acceleration due to gravity? Explain how you can tell.

   c. The driver's coffee cup, which is partially full, is in a cup holder that prevents it from spilling. The driver observes that the surface of the liquid in the cup is parallel to the floor of the car.

      In terms of the given quantities, determine an expression for the angle θ that the banked track makes with horizontal. Show all work.

4. A very tall post in Grand Rapids (north latitude $\alpha \approx 43°$) is used to support a very long plumb line. The post is oriented along an imaginary line that points directly to the center of the earth, as shown below. A plumb bob suspended from the top of the post (point *P*) is at rest relative to an observer standing next to the post. The plumb bob itself is *not* shown in the diagram.



In all parts of this problem, consider a reference frame in which the plumb bob remains <u>at rest</u> at the <u>origin</u> of that frame.

a. In the space provided, draw a free-body diagram for the plumb bob. Clearly label each force, including all physical and fictitious "forces." (Use the coordinate axes provided for reference.)

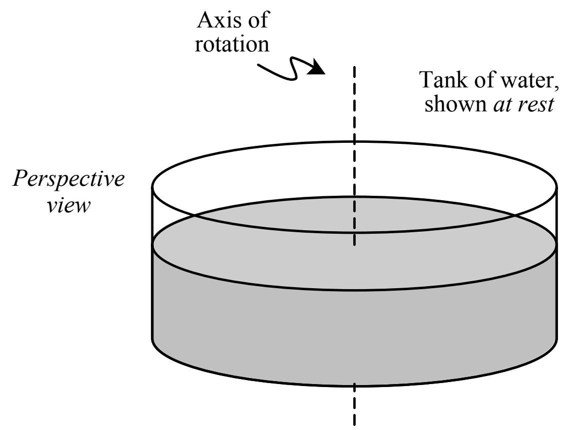
b. On the basis of your free-body diagram above, how does the *local* acceleration due to gravity compare to that of the *true* acceleration due to gravity? Discuss both magnitude and direction.

c. Suppose that the plumb bob were suspended near a calm lake. What do your answers above suggest about the orientation of the water's surface compared to (i) the orientation of the *plumb line?* (ii) the orientation of the *post?*

d. The shape of the earth is not *exactly* spherical, contrary to what the above diagram of the earth would suggest. Use your results above to explain *how* and *why* the earth's shape is different from that of a sphere.

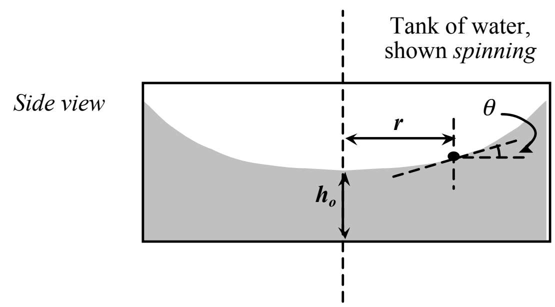
e. On the basis of your results in parts a – d of this problem, at which location(s) on earth, if any, would a stationary plumb bob point *exactly* to the center of the (non-spherical) earth? Explain.

5. Consider a cylindrical tank of water that is made to spin at a constant angular speed ω about the axis shown in the diagram. (In the second diagram below, assume that the tank has been spinning for a long time such that the water does not move with respect to the frame in which the tank is at rest.)



   a. For a location in the (spinning) tank at a horizontal distance r from the axis of rotation, express the magnitude of the local acceleration due to gravity in terms of r, ω, and g. Show all work.

   b. Use your result in part a to express the angle θ made by the surface of the water (relative to the horizontal) as a function of r. Show all work.



   c. Let h_o represent the depth of the water at the exact center of the spinning tank (see diagram). Use your result in part b to show that the surface of the water is *paraboloid* in shape. That is, if the function $h(r)$ represents the depth of the water as a function of r, show that:

$$h(r) = h_o + kr^2$$

   where k is a constant value. Show all work.

