### *Emphasis*

This tutorial is the first of a series of three on accelerating reference frames. The emphasis here is on frames whose origins move with translational acceleration relative to an inertial frame. The tutorial introduces students to and guides them to apply the concepts inertial "force" and "local acceleration due to gravity."

### *Prerequisites*

It is recommended, though not required, that students have had lecture instruction on fictitious "forces" and accelerating reference frames. This tutorial, however, may be used before such lectures as a way to introduce the subject.

## TUTORIAL PRETEST

Students consider a cart that launches a ball perpendicular to the base of the cart. The students are asked to predict in each of three possible situations whether the ball, after having launched from the cart, drops back into the launcher, falls in front of the launcher, or falls behind it. The first situation involves constant velocity along a horizontal tabletop. The second situation involves constant acceleration by using the cart to make a modified Atwood's machine. In the third (and most challenging) scenario the cart accelerates down a smooth incline (with effects due to friction and the rotation of the wheels ignored).

The first two tasks can be used to provide a baseline of student understanding of projectile motion from introductory mechanics. The third task, however, which can be solved in a relatively straightforward manner by considering the motion of the launched ball in the frame of the cart, probes the ability of students to recognize this fact and carry out the (qualitative) analysis.

## TUTORIAL SESSION

### *Equipment and handouts*

Each group will need a whiteboard and set of markers, or a large sheet of paper. Each student will need a copy of the tutorial handout (no special handouts required).

### *Discussion of tutorial worksheet*

*Section I: Inertial "force"*

In this section the terms inertial "force" and local acceleration due to gravity are motivated by considering the motion of a container of water sliding frictionlessly down a long, straight incline. The container also has attached a plumb line. For the case in which the plumb bob does not move in the rest frame of the container, students are expected to use their knowledge of the acceleration $|\vec{A}| = g\sin\theta$ of the container down the incline, the known (actual) gravitational force on the bob, and the inertial "force" $\vec{F}'_{inertial} = -m\vec{A}$ on the bob to infer that the tension force by the string must point perpendicular to the incline. Students also infer that the surface of the water inside the container, being perpendicular to the plumb line, must be oriented parallel to the surface of the incline. These results motivate the idea of local acceleration due to gravity, $\vec{g}' = \vec{g} - \vec{A}$, as a combination of the effects of the actual gravitational force and inertial "force" as viewed in the accelerating frame.

As this situation may be the first in which the students must actively analyze forces, both real and fictitious, encourage students to label clearly the forces exerted on the plumb bob, so as to distinguish which forces are real (*e.g.,* have actual agents) and which are fictitious. Students will usually have little difficulty recognizing that in the non-inertial frame of the container there are three forces (two real and one fictitious) on the plumb bob that must be add to zero, however some may stumble in drawing the free-body diagram for the bob. The "big hint" provided here to the students should help them recognize that the gravitational force and inertial "force" are both known, and from these the tension force must be inferred. At the checkpoint at the end of the section, make sure students can articulate their reasoning and give a correct explanation in comparing the magnitudes of the actual and local accelerations due to gravity in the container frame, *i.e.,* that $g'$ is less than $g$.

*Section II: Application: Launch cart on a ramp*

In this section students revisit the third task from the pretest, in which a launch cart glides down a long, straight incline. Students should be able to apply their results from section I in order to conclude that, in the absence of friction between the incline and the cart (and neglecting the rotational motion of the wheels of the cart), the ball should drop back into the launcher.

In part A students are asked to draw accurate free-body diagrams for both the cart and the launched ball as viewed in the frame of the cart, and most of the errors that arise in this section arise in the drawing of these diagrams. Ask students directly about the "hint" that is provided, because many will likely have difficulty deciding whether or not to attribute an "inertial" force to the ball after it leaves contact with the launch cart. It may help to ask students about a simpler situation, for example, observing a hummingbird that hovers above a point above the ground while viewing it from an accelerating train. Students should recognize that any object, whether or not is in contact with the launch cart (or the aforementioned train), must be attributed an inertial "force" by any observer who is at rest in the non-inertial frame.

## TUTORIAL HOMEWORK

The homework gives students the opportunity to apply and extend their results from the tutorial. Some of the applications are qualitative, others are more quantitative.

A special note about Problem 1, 3, 4, and 5: These problems involve physical situations in which the origin of an accelerating frame moves with uniform circular motion, not acceleration along a straight line, relative to an inertial frame. These problems may be assigned immediately after this tutorial because in each case the relevant object remains at rest as observed in the non-inertial frame of interest. Alternatively, these problems may be assigned after all instruction (including other tutorials) on rotating frames.

1. This problem asks students to give qualitative comparisons between actual and local acceleration due to gravity in two different non-inertial frames. One such frame accelerates in a straight line relative to an inertial frame; the other moves with uniform circular motion.
2. Students revisit the scenario from tutorial in which a launch cart glides down a long, straight ramp, however students must take into account friction between the cart and the incline.
3. Students analyze forces (real and fictitious) in the frame of a car moving with constant speed around a circular track.
4. Students examine qualitatively the forces (real and fictitious) on a plumb bob that is at rest relative to an Earthbound reference frame. They infer from their results how the shape of the Earth must differ from that of a perfect sphere.
5. Finally, students apply the concept of local acceleration due to gravity to determine quantitatively the shape of the surface of a liquid in a cylindrical container spinning at constant angular speed.