1. (*Note: This problem also serves as a post-test for* Energy and angular momentum for closed orbits.)

The diagram at right shows three closed orbits *(1, 2,* and *3)* around a planet of radius *R* and mass *M.* Orbit *2* is *circular,* and orbits *1* and *3* are elliptical orbits having the *same eccentricity .* All three orbits are tangent to one another at the point labeled *P* in the diagram.

A. For a satellite that follows orbit *1,* is the speed of the satellite at point *P* *greater than, less than,* or *equal to* that at point *Q?* Explain.

B. From greatest to smallest, rank the three orbits *(1, 2,* and *3)* according to:

 i. the semimajor axis of the orbit. Explain how you can tell.

ii. the total energy of a satellite following that orbit. Explain.

C. Suppose that the eccentricity of orbit *1* (and orbit *3*) is ** = 0.5, and that point *P* is a distance **6*R*** from the center of the planet. In terms of *G* (Newton’s gravitation constant), *M,* and *R,* determine expressions for:

i. the speed of a satellite passing point *P* along orbit *1.* Show all work.

ii. the speed of a satellite passing point *Q* along orbit *1.* Show all work.

2. (*Note: This problem also serves as a post-test for* Energy and angular momentum for closed orbits.)

Three identical shuttlecraft follow three closed orbits *(1, 2,* and *3)* around a planet.

Orbits *2* and *3* are circular. Orbits *1* and *3* have *equal* semi-major axes *(a1* = *a3).*

A. Rank the three orbits according to each of the following quantities. Briefly explain your reasoning in each case.

i. eccentricity

ii. total energy

iii. angular momentum

iv. period of orbit

B. For the shuttle that follows orbit *1,* is the speed of the shuttle at point *X* *greater than, less than,* or *equal to* that at point *Y?* Explain how you can tell.

C. Along which orbit would a shuttle have the larger speed at **point *X:*** *orbit 1, orbit 2,* or would it be *the same* in either case? Explain how you can tell.

D. Along which orbit would a shuttle have the larger speed at **point *Y:*** *orbit 1, orbit 3,* or would it be *the same* in either case? Explain how you can tell.

E. If the latus rectum of orbit *1* is half that of orbit *3* *(i.e.,* *1* = ½*3),* determine the angle formed by orbits *1* and *3* where they intersect at point *Y.* Show all work.

3. (*Note: This problem also serves as a post-test for* Energy and angular momentum for closed orbits.)

Your shuttlecraft (mass *m*) is orbiting a planet (mass *M,* radius *R)* in a circular orbit of radius 9*R* when the retrothrusters suddenly misfire, causing your shuttle’s speed to suddenly drop to:



*Note:* In this problem, you may express your final answers in terms of *m, M, R,* and *G.*

A. In terms of the given parameters, determine (i) the total energy of the planet-shuttlecraft system and (ii) the angular momentum of the shuttle measured with respect to the center of the planet. Clearly show all work.

B. Determine the semimajor axis of your new (elliptical) orbit, and in so doing, show that your new orbit will not cause you to crash into the planet. Clearly show all work.

C. At the location when your new orbit causes to pass a point located 4*R* away from the center of the planet, determine:

• the speed of your shuttle as it passes that point

• the angle ** of approach of your shuttle as it passes that point (see diagram at the top of the page)

4. (*Note: This problem also serves as a post-test for* Energy and angular momentum for closed orbits.)

Three identical satellites follow three different orbits around the Earth, as shown. Orbits *1* and *2*are circular; orbit *3* is elliptical. All orbits are *drawn to scale,* with each grid on the diagram being one Earth-radius (*R*E) on a side. (*Note:* Ignore interactions between satellites.)

A. Rank the three orbits according to each of the following quantities. Briefly explain how you determined each ranking.

i. eccentricity

ii. orbital period

iii. angular momentum

B. Imagine that the satellite along orbit *1* suddenly decreased its speed, causing its new orbit to *barely miss* colliding with the Earth. (That is, its new perigee is exactly 1*R*E.)

 Determine expressions for the quantities listed below. Wherever appropriate, your answers should be in terms of physical constants (such as *R*E, *G,* and *M*E, the mass of the Earth). **Clearly show all work.**

• the semi-major axis of the new orbit

• the eccentricity ** of the new orbit

• the value to which the speed of the satellite suddenly dropped (causing this near disaster to occur in the first place)

5. (*Note: This problem also serves as a post-test for* Energy and angular momentum for closed orbits.)

Red alert! While orbiting a planet (mass *M,* radius *R)* a satellite experiences a malfunction with its navigation systems. At a point 4*R* from the center of the planet it assumes an approach angle of 55° (see diagram) with speed equal to:



A. Show that the satellite’s velocity at its current location will not allow it to “escape” the planet (*i.e.,* show that its new orbit will be closed rather than open).

B. In terms of *R,* calculate the following features of the satellite’s orbit. Show all work.

• semi-major axis

• latus rectum

C. Will the satellite *crash* into the planet or will it *miss* the planet? Explain your reasoning and show your work.

6. (*Note: This problem also serves as a post-test for* Conservative force fields.)

A small plastic puck is placed on a frictionless horizontal air table. The puck is made to move under the influence of a (net) force expressed in Cartesian coordinates as:

*,* where *c* is a constant.

A. For what value of *c* is the above force conservative? Show your work.

B. Is the above force separable? Explain how you can tell and/or show your work.

C. As the puck moves under the influence of the above force, would the angular momentum of the puck (with respect to the origin, (0, 0)) be conserved? Explain how you can tell and/or show your work.