INSTRUCTOR NOTES
Damped oscillations: Motion graphs

Emphasis

Students examine qualitative aspects of the motion of underdamped oscillators. They investigate the effect of damping on the frequency of an oscillator. In addition, students are guided to recognize how the qualitative time-dependence of the velocity of an underdamped oscillator differs from that of an ideal simple harmonic oscillator.

Prerequisites

Students need to have completed instruction of underdamped oscillations. Completion of the tutorials *Simple harmonic motion* and *Newton's laws and velocity-dependent forces* is recommended but not required for this tutorial.

TUTORIAL PRETEST

The pretest probes student understanding of two important features of the motion of an underdamped oscillator: (i) the amplitude diminishes with time, and (ii) the frequency, though constant, is smaller than the natural frequency (*i.e.,* the frequency of the oscillator in the absence of damping). The students are also asked whether the underdamped oscillator is speeding up, slowing down, or moving with (momentarily) constant speed as it passes through $x = 0$.

Students often recognize that the amplitude decreases exponentially with time. However, many incorrectly sketch the *x vs. t* graph with the same frequency as the undamped case (*i.e.,* the natural frequency) case, even though it may have been proven in lecture that the frequencies must be different according to the relationship $\omega_d = (\omega_o^2 - \gamma^2)^{1/2}$. Some students may instead believe that there is no single value for the frequency of an underdamped oscillator, thinking instead that each cycle of the motion transpires over a time interval that is smaller than that for the preceding cycle.

Many students fail to recognize that upon passing $x = 0$ the oscillator experiences a net force (due to damping) opposite in direction from its velocity. Rather than state that the oscillator is slowing down at $x = 0$ students incorrectly state that the maximum speed is attained at $x = 0$. This error, along with the failure to recognize the effect of damping on the frequency, suggests that students inappropriately make generalizations from the special case of undamped (frictionless) oscillators when they attempt to make sense of damped oscillators.

TUTORIAL SESSION

Equipment and handouts

Each group will need a whiteboard and set of markers, or a large sheet of paper. Each student will need a copy of the tutorial handout (no special handouts required).

Discussion of tutorial worksheet

Section I: Position versus time

Students are first shown a displacement versus time graph for an underdamped oscillator that is released from rest. They are expected to interpret from the graph that the frequency is well-defined even though the amplitude gradually decreases with time. Students are then asked to consider the case in which the oscillator is released from rest at the same location as before but with the damping removed. They reason that, without damping, the oscillator will take less time than before to reach $x = 0$. This result suggests that the period of the (undamped) oscillator will be smaller than that of the damped oscillator. Students then conclude that the presence of the damping force will decrease the frequency of an oscillator. Students should use the "stepping stone" questions in the tutorial to construct their argument rather than the mathematical relationship $\omega_d = (\omega_o^2 - \gamma^2)^{1/2}$.

(*Note:* It is true that, for any underdamped oscillator starting from rest, *slightly more than* one quarter of the period transpires between the instant of release and the instant the oscillator crosses $x = 0$. Bright students who recognize this fact may believe there is insufficient information to conclude that the period is made smaller by removing the damping force. It may help to encourage such students to think about extreme cases: Have students imagine that the damping were quite substantial—which is in fact true for the motion shown in the original graph [the ratio of successive maxima is only 0.5]—and then have students imagine the effect of completely removing such a damping force.)

Section II: Velocity versus time

In this section of the tutorial students revisit the pretest question in which they were asked about the motion of the underdamped oscillator as it passes through $x = 0$. The students are asked to draw qualitatively correct *x vs. t* and *v vs. t* graphs for an underdamped oscillator. (*Note:* If students draw qualitatively incorrect *v vs. t* graphs on their first attempt, let them continue until the checkpoint at the bottom of page 3.)

Students then consider the forces exerted on the oscillator at the moment it crosses $x = 0$. In particular, they are guided through the reasoning to recognize that at such a moment the oscillator experiences a net force o velocity. (The restoring force by the spring is zero, so the net force is equal to the damping force.) This result means that the oscillator is always slowing down from the moment it crosses $x = 0$ until the moment it reaches the next turnaround point. The maximum speed attained during each half-period of motion must therefore occur before the oscillator reaches $x = 0$. Students use this conclusion to check (and, if necessary, correct) the *v vs. t* graph they drew originally.

TUTORIAL HOMEWORK

Of the four homework problems provided for this tutorial, two deepen and extend the connections between the physics of underdamped oscillators and motion graphs. The other two focus on connections between the physics and mathematical expressions for *x(t)* and *v(t)*.

1. Students compare the motion of underdamped oscillators that have the same ω_d and different values of γ (part a) or the same γ and different values of ω_d (part b). Part c of this problem provides students the opportunity to apply and extend their result from tutorial that the presence of a damping force lowers the frequency of an oscillator.

2. Students show that, using $x(t) = Ae^{-\gamma t}\cos(\omega_d t + \phi_o)$ to represent position as a function of time, an oscillator starting from rest must be described by a nonzero value of ϕ_o.

   In addition, even though the instants when the cosine function has value +1 or –1 are *not* the same instants when the oscillator achieves maximum displacement, students show that the oscillator attains maximum displacements at instants that are $T_d/2$ apart.

3. Students prove mathematically—as they will have done by qualitative arguments during the tutorial—that an underdamped oscillator released from rest will first attain a maximum speed *before* reaching $x = 0$.

4. Students extend results from section II of the tutorial by drawing qualitatively correct graphs of kinetic energy, potential energy, and total energy of an underdamped oscillator as functions of time. Students are expected to recognize that the total energy is always decreasing *except* at the turnaround points. At these moments the velocity is zero and thus no work is being done on the oscillator by the damping force. Therefore, it is *incorrect* to say that the instantaneous total energy decreases exponentially with time (as asked in part d), however the *average energy* per oscillation may be said to decrease exponentially with time.

