1. (*Note: This problem may also serve as a post-test for* Damped harmonic motion: Motion graphs.)  
  
Consider two linear, underdamped oscillators (#1 and #2) that are identical in *every* way except for the magnitudes of their damping constants *(1* ≠ *2).* The following information is known:

• Oscillator #1 (whose undamped frequency *o* is equal to that of oscillator #2) has a damped frequency **d1 = 0.995 *o*.

• Oscillator #2 has a quality factor *Q2* = 8.0

A. On the basis of the information above:

• Which oscillator (#1 or #2) has the larger *damping constant?*

• Which oscillator has the larger (damped) *frequency?*

• Which oscillator has the larger *quality factor?*

Justify your answers with appropriate calculations and explanations.

B. If the undamped frequency of oscillator #1 is *o* = 1.6 s-1 (and **d1 = 0.995 *o*), determine the ratio of the amplitudes of two successive maxima of that oscillator. Show all work.

2. Two different damped oscillators (#1 and #2) are released from rest at *t* = 0. The graphs below illustrate the motion of the oscillators.



(*Note:* Do **not** assume that the natural frequencies of the oscillators are the same.)

A. Is the damping constant *(1)* of oscillator #1 *greater than, less than,* or *equal to* the damping constant *(2)* of oscillator #2? Explain how you can tell.

B. Is the quality factor of oscillator #1 *greater than, less than,* or *equal to* that of oscillator #2? Explain how you can tell.

3. (*Note: This problem may also serve as a post-test for* Damped harmonic motion: Motion graphs.)  
  
Two damped oscillators (#1 and #2) are released from rest from different locations at *t* = 0, as shown below right. Oscillator #1 is released from *x* = 0.75 m; oscillator #2, from *x* = 1.00 m. Note that at *t* = 1.5 s both oscillators experience maximum displacement at the same location, *x* = 0.25 m.

(*Note:* Do **not** assume that the natural frequencies of the oscillators are the same.)

A. Is the damping constant *()* of oscillator #1 *greater than, less than,* or *equal to* that of oscillator #2? Explain how you can tell.

B. Is the quality factor of oscillator #1 *greater than, less than,* or *equal to* that of oscillator #2? Explain how you can tell.

C. Use the information provided in the graph for oscillator #1 to evaluate the constants *a* and *b* in its differential equation of motion (see below). Clearly show all work.



4. (*Note: This problem may also serve as a post-test for* Phase space diagrams: Damped harmonic motion.)  
  
A harmonic oscillator with mass *m* and undamped angular frequency *o* is subject to a damping force *mo**.* (Assume that all constants are positive.)

A. Write down the differential equation that governs the motion of this oscillator.

B. What can be said about the value of the parameter *o* if the above oscillator were (i) critically damped? (ii) underdamped?

C. If the quality factor of the above oscillator is equal to *Q* = 15, then determine each of the following quantities in terms of *o* and numerical constants. Clearly show all work. [*Hint:* Do *not* solve the differential equation if you don’t have to!]

i. the period **d of the damped oscillator

ii. the period *o* of the oscillator when the damping is removed

iii. the ratio of the amplitudes of two successive maxima of the damped oscillator



D. On the basis of your results above, carefully sketch a qualitatively correct phase space plot for the first cycle of the motion of the oscillator, starting at point *P.*

*(Note:* Shown for reference is the trajectory that would represent the motion of the oscillator *if* the damping were *removed.)*

5. (*Note: This problem may also serve as a post-test for* Forced harmonic motion.)  
  
A mass *m* = 400 g is connected to a spring with spring constant *k* = 1.6 N/m. The oscillator is subject to a retarding force proportional to the velocity of the oscillator, with damping constant ** = 0.6 s-1.

A. Write down the differential equation of motion for the damped oscillator, using the numerical values of given parameters *(m, k, )* as appropriate. (*Note:* Do **not** bother to solve the equation.)

B. Determine the angular frequency of the damped oscillator. Show all work.

C. Determine the ratio of the amplitudes of two successive maxima of the damped oscillator. Show all work.

D. Determine the angular frequency of an external driving force required for the (damped) oscillator to achieve resonance. Show all work.