INSTRUCTOR NOTES
Damped oscillations: Energy loss and the quality factor

Emphasis

Students examine qualitatively and quantitatively the time-dependence of the amplitude of an underdamped oscillator as well as the quality factor for such an oscillator.

Prerequisites

Students need to have completed instruction of underdamped oscillations. It is recommended that students have familiarity with the concept of quality factor as well as the relationship $Q = \omega_d/2\gamma$ for weakly damped oscillators. Completion of the tutorial *Damped oscillations: Motion graphs* is recommended but not required for this tutorial.

TUTORIAL PRETEST

The first pretest question probes student understanding of the relationship between energy and amplitude. The information in the problem statement describes an underdamped oscillator whose amplitude decreases to a value 80% as large with each oscillation. The students are expected to recognize—but often do not—that the energy must decrease to a value 64% as large after each oscillation, due to energy being proportional to the square of the amplitude.

The second question asks students to predict the amplitude of the oscillator upon completing two full oscillations. They are expected to recognize that the ratio of successive amplitudes is a constant value, so that the maximum displacement at the end of the second oscillation is 0.64 m. Some students may incorrectly say that this maximum displacement is 0.60 m, thinking that successive amplitudes differ by the same decrement (0.20 m) rather than by the same ratio (4/5).

TUTORIAL SESSION

Equipment and handouts

Each group will need a whiteboard and set of markers, or a large sheet of paper. Each student will need a copy of the tutorial handout (no special handouts required).

Discussion of tutorial worksheet

Section I: Amplitude of underdamped oscillations

In part A, students consider an underdamped oscillator that begins from rest at $t = 0$. A table provides the initial displacement and the maxima at each of the next three cycles. Students are asked to look for a pattern in the data and are expected to discover that the ratio of successive maxima is always a constant value. In part B, students show that this pattern must arise on the basis of the solution $x(t) = Ae^{-\gamma t}\cos(\omega_d t + \phi_o)$ to the equation of motion for the underdamped oscillator. Specifically, students are guided to recognize that the quantity $\exp(-\gamma T_d)$ is equal to the ratio of successive maxima.

(*Note:* Students do not need to, and therefore should not, make the approximation that $\phi_o \cong 0$ here. They instead need to recognize that the values of $\cos\phi_o$ and $\cos(2\pi + \phi_o)$ are equal, regardless of the value of ϕ_o. In a homework problem from the preceding tutorial, *Damped oscillations: Motion graphs,* students realize that $\phi_o \cong 0$ is an approximation suitable for an underdamped oscillator released from rest only for the case of weak damping.)

Section II: Quality factor

In this section of the tutorial the students are introduced to and gain practice with the concept of the quality factor *Q,* which is defined to be equal to 2π divided by the fraction of energy dissipated during any given cycle. In part A students are guided step by step through a derivation of Q: letting *"r"* denote the ratio of successive maxima ($r = \exp(-\gamma T_d)$), students should recognize that the quality factor must be equal to 2π divided by the quantity $(1 - r^2)$. In part B students apply their result by calculating Q for the damped oscillator described in section I. They also use motion graphs to illustrate changes in oscillation frequency or quality factor, with the goal of reinforcing with students the one-to-one correspondence between quality factor and the ratio of successive maxima. Students conclude the tutorial in part C by rewriting their expression for Q in terms of γ and T_d (instead of r):

$$Q = \frac{2\pi}{1 - e^{-2\gamma T_d}}$$

TUTORIAL HOMEWORK

The homework gives students the opportunity to apply and extend their results from the tutorial, both qualitatively and quantitatively.

1. Students review and apply results obtained from the tutorial—the constancy of the ratio of successive maxima, the relationship between maximum amplitude and energy, and the definition of the quality factor.
2. In part a of this problem students must recognize how changing the damping constant γ affects the frequency ω_d of the oscillator. (This relationship is emphasized in the preceding tutorial, *Damped oscillations: Motion graphs*.) In part b students must also give a qualitative interpretation to one of their results from tutorial, specifically how the value of the quality factor Q depends on both γ and ω_d (or T_d).
3. Students examine the special category of underdamped oscillators for which $\gamma T_d \ll 1$. After justifying the term *weakly damped* for such an oscillator, students use the power series expansion of the exponential function to derive the more familiar expression $Q \cong \omega_d/2\gamma$ that applies to weak damping.
4. Students extend their result $Q \cong \omega_d/2\gamma$ for weakly damped oscillators by considering such an oscillator whose amplitude decreases by a factor of e after every N cycles. They reason qualitatively that it makes more sense that Q should be proportional to N (rather than inversely proportional to or independent of N) and then prove the relationship $Q = N\pi$.
4. Students compare the motion of underdamped oscillators that have the same damping constant γ and different values of quality factor Q (part a) or the same Q and different values of γ (part b). These exercises are similar to those in section II.B of the tutorial, except these are designed to probe how well students can differentiate the effects of varying γ (which describes how quickly the amplitude decreases *per unit time*) and Q (which describes how quickly the amplitude decreases *per oscillation*).

