CAUCHY'S THEOREM AND CURVE DEFORMATION

Directions: Work in groups of two to four students to answer the questions below. Each group will submit one neatly handwritten copy of its solutions on the indicated due date.

Let a be an arbitrary complex number, and consider the contour integral

$$\int_{|z-a|=2} \frac{z}{(z-(-1+i))(z+i)}\,dz\,.$$

We wish to investigate how the value of this contour integral depends upon a.

1. Suppose first a=0. Explain, for this choice of a, why the above integral is the same as the sum of the two integrals

$$\int_{|z-(-1+i)|=.2} \frac{z}{(z-(-1+i))(z+i)}\,dz + \int_{|z+i|=.2} \frac{z}{(z-(-1+i))(z+i)}\,dz\,.$$

   Hint: Use the idea of curve deformation.

2. Determine the partial fractions decomposition of the function $\frac{z}{(z-(-1+i))(z+i)}$. In other words, find complex numbers A and B that satisfy

$$\frac{z}{(z-(-1+i))(z+i)} = \frac{A}{z-(-1+i)} + \frac{B}{z+i}\,.$$

3. Use your results from (1) and (2), together with the Integration of Powers Theorem and and Cauchy's Theorem, to evaluate

$$\int_{|z|=2} \frac{z}{(z-(-1+i))(z+i)}\,dz\,.$$

4. Repeat the reasoning above in order to evaluate

$$\text{A. } \int_{|z-1|=2} \frac{z}{(z-(-1+i))(z+i)}\,dz \quad \text{and} \quad \text{B. } \int_{|z-2|=2} \frac{z}{(z-(-1+i))(z+i)}\,dz$$

5. Now view the **f(z)** file "curvedef.fzw." This file plays out an animation of what happens to the integral $\int_{|z-a|=2} \frac{z}{(z-(-1+i))(z+i)}\,dz$ as a increases continuously from 0 to 2. In Figure 1 you will see the circle $|z-a|=2$, in Figure 2 the image of this circle under the map $f(z)=\frac{z}{(z-(-1+i))(z+i)}$, and in Figure 3 an arc that starts at the origin and ends at the value of the contour integral. How many different values are there for the contour integral as a increases from 0 to 2? For each different value of the contour integral, which values of a correspond to that particular value?