Euler's Identity, the Complex Exponential, and the Polar Form Revisited

Math 402, Winter 2001

Directions Work in groups of three to four students on this activity.
The goal is to derive and investigate *Euler's identity*:

$$e^{i\theta} = \cos(\theta) + i\sin(\theta), \quad \theta \in \mathbb{R}.$$

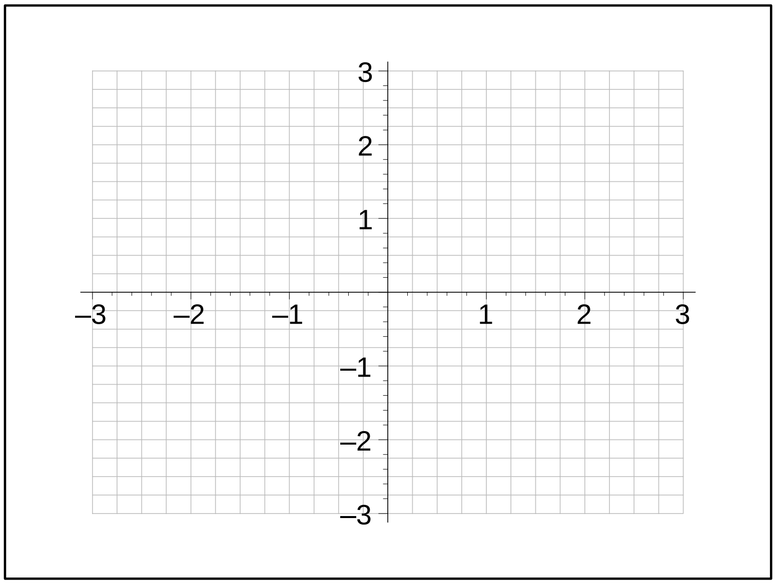
1. Write down the Maclaurin series for each of the functions, e^x, $\sin(x)$, and $\cos(x)$.

2. Substitute the quanity $x = i\theta$ into the Maclaurin series for e^x. Simplify all powers of i.

3. Rearrange the terms in the series obtained in the previous problem, and use the Maclaurin series for $\sin(x)$ and $\cos(x)$ to derive Euler's identity.

4. Consider the quantity e^{2i}. Use the first six terms of the series from (2) to approximate this quanity. Simplify each of the six terms, but do not add them together yet.

5. On the axes below, draw the first term in the above sum as a vector with initial point at the origin. Then draw the second term in the sum as a vector with initial point at the terminal point of the first vector, draw the third term in the sum as a vector with initial point at the terminal point of the second vector, and so on. Use vector addition, together with the grid, to estimate e^{2i}.



6. Now use your graphing calculator to estimate e^{2i} two ways. For one way, use Euler's identity. For the other way, simply enter $\mathbf{e} \wedge (\mathbf{0}, \mathbf{2})$.

7. Use Euler's identity to explain why every complex number z can be written, not necessarily uniquely, in the form $z = re^{i\theta}$.