MAPPING PROPERTIES OF COMPLEX-VALUED FUNCTIONS
MATH 402, WINTER 2001

In this activity we will investigate mapping properties of certain common complex-valued functions. You will work in groups, with each group studying a particular function, generating a single write-up, and providing the class with a brief verbal summary of the group's function mapping properties. Each group should submit both a hard copy of its report as well as an electronic version on disk in either Microsoft Word for Windows 98 or in LaTex format. **F(z)** images may be imported using the **copy to bitmap** command found under the **edit** menu. A collated document containing all groups' findings will eventually be distributed to all students.

Your group is assigned the function $f(z) =$ __________________.

1. What is the domain of your function?

2. If $f(z) = f(x+iy) = u(x,y) + i \cdot v(x,y)$, determine the real part u and imaginary part v of your function.

3. Determine exact rectangular coordinates for the following outputs of your function:
   A. $f(1+2i)$
   B. $f(-1-3i)$

4. Experiment with various sets that I assign your particular group. Sketch these sets, and use **F(z)**, together with your knowledge of complex numbers, to determine the image of these sets under your function. Label interesting curves and points in the domain and their corresponding images in the range so that one can clearly determine which values in the input set are mapped to which values in the output set. Give a clear, concise geometric explanation of what your function does to an input value $z = x + iy = re^{i\theta}$ to obtain the output value $f(z)$. Then write a sentence or two that summarizes your findings.

5. What is the range of your function?

6. For each value $w = f(z)$ in the range of your function, how many inputs z are mapped to w? Are these input values arranged in any interesting geometric manner?

7. Is your function one-to-one? Does it have an inverse function?