

Mapping Properties of Complex-Valued Functions

List of Functions and Sets

Math 402, Winter 2001

1. $f(z) = e^{2i}z$

(a) $|z| \leq 2$ and $\text{Arg}(z) = \frac{k\pi}{3}$, where $k = 0, 1, 2, 3, 4, 5, 6$.

(b) Now consider in succession the functions $f(z) = e^{\alpha i}z$, $f(z) = 3e^{\alpha i}z$, and $f(z) = 3e^{\alpha i}z + (1 + i)$ applied to the set in (a). The quantity α is assumed to be real.

2. $f(z) = z^2$

(a) $|z| \leq 2$.

(b) $|z| \leq \frac{1}{3}$.

(c) $|z| \leq 3$ and $0 \leq \text{Arg}(z) \leq \pi$.

(d) $|z| = 2$ and $\frac{\pi}{4} \leq \text{Arg}(z) \leq \frac{3\pi}{4}$.

3. $f(z) = z^3$

(a) $|z| \leq 2$.

(b) $|z| \leq \frac{1}{3}$.

(c) $|z| \leq 3$ and $0 \leq \text{Arg}(z) \leq \frac{\pi}{2}$.

(d) $|z| = \frac{1}{4}$ and $\frac{\pi}{6} \leq \text{Arg}(z) \leq \frac{\pi}{5}$.

4. $f(z) = \frac{1}{z}$

(a) $|z| \leq 2$.

(b) $|z| \geq 3$.

(c) $\frac{1}{2} \leq |z| \leq 3$ and $\text{Arg}(z) = \frac{2\pi}{3}$.

(d) $|z| = 3$. Describe the orientation of the image of this circle.

5. $f(z) = e^z$.

(a) $S = \{z = x + iy \mid 0 \leq x \leq 2, 0 \leq y \leq \pi\}$.

(b) $S = \{z = x + iy \mid -3 \leq x \leq 2, -\pi \leq y \leq \pi\}$.

(c) $S = \{z = x + iy \mid x \leq 2, -\pi \leq y \leq \pi\}$.

(d) $S = \{z = x + iy \mid x \leq 2, -2\pi \leq y \leq 2\pi\}$.

(e) $S = \{z = x + iy \mid x = 3, 0 \leq y \leq 2\pi\}$.

(f) $S = \{z = x + iy \mid -\frac{1}{2} \leq x \leq 3, y = 1\}$.