

HOW TO SOLVE IT

George Pólya (1957)

“Thus, a teacher of mathematics has a great opportunity. If he fills his allotted time with drilling his students in routine operations he kills their interest, hampers their intellectual development, and misuses his opportunity. But if he challenges the curiosity of his students by setting them problems proportionate to their knowledge, and helps them to solve their problems with stimulating questions, he may give them a taste for, and some means of, independent thinking.”

–Georg Polya, from *How To Solve It*

Georg Polya was a mathematician, who realized that perhaps the goal of math education should be problem solving. But then we should know what problem solving is... so he studied mathematicians to find out how they solved problems.

Polya's famous four phase model of problem solving is presented below with the questions that he asked either implicitly or explicitly in *How to Solve It*.

Understanding the problem

1. What problem(s) are we trying to solve? What is the unknown? What is the data?
2. What are the conditions? Is it possible to satisfy the conditions? Are the conditions sufficient to solve for the unknown? Are there extra conditions? Are there contradictory conditions?
3. Can you draw a figure? Is notation an issue?
4. Can you separate the parts of the conditions?

"If there is a problem you can't solve, then there is an easier problem you can't solve: find it."

“My method to overcome a difficulty is to go round it.”

Devising a plan

1. Have you seen this problem before? Have you seen it in a different form?
2. Do we know a related problem? Do you know a theorem that could be useful?
3. Do you know a problem with a similar unknown?
4. How can you use a problem you've already solved? Can you use its result? Can you use its solution method? Can you vary the conditions to make it usable?
5. Can you restate the problem? In multiple ways? Would revisiting definitions help?
6. Can we solve a simpler problem? A more special problem? What conditions would we need to change?
7. Could you collect information from the data? Is there other relevant data?
8. Can you vary the unknowns or the data to strengthen their connection?
9. Have you looked at all the data or used all the conditions?
10. Have you considered the essential notions in the problem?

“There are many questions which fools can ask that wise men cannot answer.”

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CARRYING OUT THE PLAN

1. What steps are being taken?
2. Have you checked each step?
3. Can you see clearly the steps are correct?
4. Can you prove each step is correct?

“A GREAT discovery solves a great problem, but there is a grain of discovery in any problem.”

“Even fairly good students, when they have obtained the solution of the problem and written down neatly the argument, shut their books and look for something else. Doing so, they miss an important and instructive phase of the work.”

LOOKING BACK

1. What is my solution to the problem?
2. Can you check that it is a solution?
3. Can you check that the argument is correct?
4. What was my method?
5. What other methods might work to solve this problem?
6. Why did I select this particular method?
7. Can you use the result for another problem?
8. Can you use the method for another problem?



“Wishful thinking is imagining good things you don't have...[It] may be bad as too much salt is bad in the soup and even a little garlic is bad in the chocolate pudding. I mean, wishful thinking may be bad if there is too much of it or in the wrong place, but it is good in itself and may be a great help in life and in problem solving.”