The Optimal Origami Box

Math in Action

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A hands-on activity where students fold origami boxes with varying heights to explore modeling and optimization. You can download this activity from my website and modify it for use in your class.

Adapted from **Unfolding Mathematics with Origami Boxes**, by Arnold Tubis and Crystal Mills.

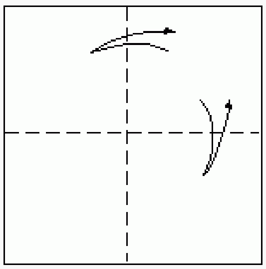
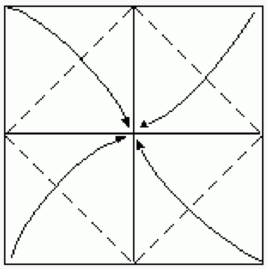
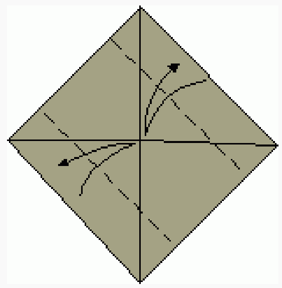
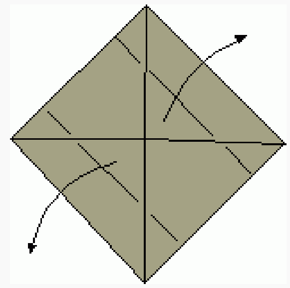
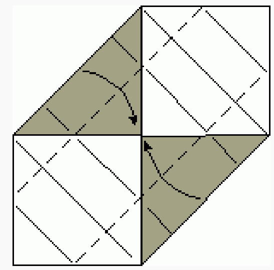
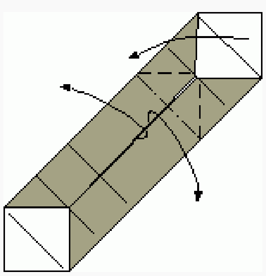
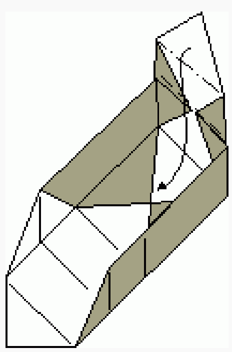
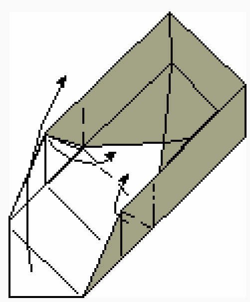
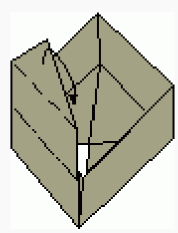
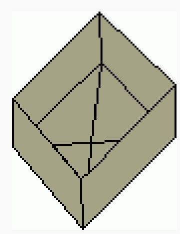
Materials:

- folding instructions: Wikipedia, Japanese masu
- square origami (scrapbooking) paper
- rulers
- graphing calculators

Folding diagram from Wikipedia entry for **Japanese masu**

http://en.wikipedia.org/wiki/Masu_%28Japanese%29

The height will be half of the length of the base. To create a box with a different height, modify steps 3 and 5 by making folded sides larger or smaller.

Step 01	**Step 02**	**Step 03**	**Step 04**	**Step 05**
				
Crease and Return	Fold tops to centre, this is called a blintz fold after a Jewish pastry	Fold sides to centre and return	Open two corners	Fold sides to centre
Step 06	**Step 07**	**Step 08**	**Step 09**	**Step 10**
				
Lift both sides and one end of the model so it becomes 3D	Fold flap to centre	Raise end	Fold flap to centre	Complete

The Optimal Origami Box **Smith**

In this activity, we will fold origami boxes with varying height, and determine which height will give us a box with the largest possible volume.

Folding Boxes and Using Data to Create a Model

1. Using your $12'' \times 12''$ square sheet of paper, fold a box following the instructions given. Note that since our paper is square, the base of the box is also square. (What does this tell you about the length and width of the box?) Measure your box and calculate its volume. What are the units on your answer?

2. Suppose we decrease the height of the box. What do you predict would happen to the length and width of the box? What about the volume of the box?

3. Suppose we increase the height of the box. What do you predict would happen to the length and width of the box? What about the volume of the box?

4. In your group, fold boxes different heights, and calculate the volume of each to fill in the following table. (Two rows are left blank for data for boxes with zero volume.)

Height	Length	Width	Volume
1			
1.5			
2.125			
2.5			

5. Enter the data from your table into your calculator to create a scatter plot of the data and sketch it below.

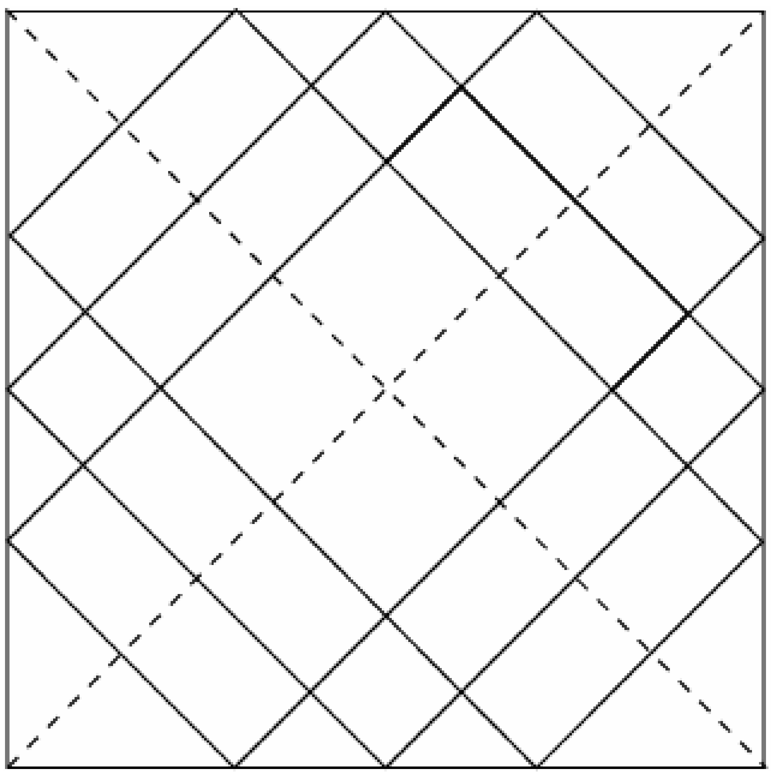
6. What type of model do you choose to approximate your scatter plot? Write the function below and graph your model on the same set of axes with the scatter plot. Does it seem like a reasonable approximation of your data? If not, can you choose a more accurate model?

7. Use your model from question 6 to determine the height should we use to maximize the volume of an origami box. What is the maximum possible volume for your origami box?

Creating a Theoretical Model

8. What is the general formula for the volume of a box with a rectangular base? This formula has too many variables, but already we can eliminate one of them because our box has a square base. What is the modified formula?

9. Unfold the boxes that you made so that we can see the creases. They should look similar to the diagram below. Trace the creases that outline the base of your box and the creases that outline the sides. What is the length of the dashed diagonal lines that have been added?



10. Identify segments of the diagonal lines on the crease diagram that can be used to measure the length and height of the box. Use these segments to find an equation that relates the length of the diagonal to the height, H, and length, L, of the box. (You may want to include the equation $\_\_\_\ L + \_\_\_\ H = \_\_\_$.) How can we use this equation to eliminate another variable from the volume formula?

11. We now have a formula for the volume of an origami box as a function of its height.
(For algebra) Graph the volume function on your calculator and find the optimal height and maximum volume of an origami box. How does this compare to your model from the data?
(For calculus)
a) Differentiate the volume function and find its critical points.

b) Which critical point is the optimal height of your origami box?

c) What is the maximum volume for your origami box?

d) How do these results compare to your results in question 7?

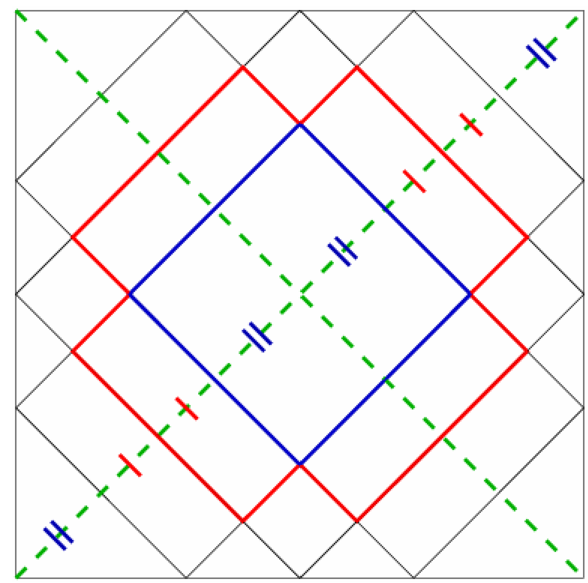
What if we change the size of the paper that we use?

12. If we change the size of the paper, how would that change the equation that you found in question 10 and the resulting volume function?

13. (For algebra) Find the optimal height and maximum volume of an origami box folded from an $8'' \times 8''$ square sheet of paper. (For calculus) Find the optimal height and maximum volume of an origami box folded from a square sheet of paper with sides *S* inches long. Remember that *S* is a constant when you are creating and differentiating your volume function, and the input variable is *H*.

Instructor notes

Crease diagram for unfolded box



Diagonal = 2·*Length* + 4·*Height*

Using $12'' \times 12''$ square paper:

$$D = 2L + 4H$$

$$\begin{aligned} L &= \tfrac{1}{2}D - 2H \\ &= 6\sqrt{2} - 2H \end{aligned}$$

$$\begin{aligned} V(H) &= H\left(6\sqrt{2} - 2H\right)^2 \\ &= 4H^3 - 24\sqrt{2}H^2 + 72H \end{aligned}$$

$$\begin{aligned} V'(H) &= 12H^2 - 48\sqrt{2}H + 72 \\ &= 12\left(H - 3\sqrt{2}\right)\left(H - \sqrt{2}\right) \end{aligned}$$

max volume is $32\sqrt{2}$ in^3 for $H = \sqrt{2}$ in.

Using $S'' \times S''$ paper:

$$\begin{aligned} L &= \tfrac{1}{2}D - 2H \\ &= \tfrac{\sqrt{2}}{2}S - 2H \end{aligned}$$

$$\begin{aligned} V(H) &= H\left(\tfrac{\sqrt{2}}{2}S - 2H\right)^2 \\ &= 4H^3 - 2\sqrt{2}SH^2 + \tfrac{1}{2}S^2H \end{aligned}$$

$$\begin{aligned} V'(H) &= 12H^2 - 4\sqrt{2}SH + \tfrac{1}{2}S^2 \\ &= 12\left(H - \tfrac{\sqrt{2}}{12}S\right)\left(H - \tfrac{\sqrt{2}}{4}S\right) \end{aligned}$$

max volume is $\tfrac{\sqrt{2}}{54}S^3$ in^3 for $H = \tfrac{\sqrt{2}}{12}S = \tfrac{1}{12}D = \tfrac{1}{4}L$