

Description of the Portfolio Project

During the semester, twelve problems will be posed to all students. Each student will work on these problems and submit proposed solutions to these problems to the professor by the end of the semester. You may submit drafts of solutions to each of the portfolio problems to the professor two times to be critiqued. The professor will review and critique your submission, make recommendations and indicate whether or not your submission is ready for the portfolio or not.

Electronic Submission of Portfolio Problems

Each solution or proof must be done on a word processor capable of producing the appropriate mathematical symbols and equations. Microsoft Word and its Equation Editor, which is available on the student network, is one such word processor.

Each solution or proof for a portfolio problem must be submitted to the instructor electronically as an attachment to an email message, which must be sent to the following email address:

mth210.w08@gmail.com

The file name for your attachment must be in the following format:

LastName Problem n Draft m .doc

For example, if Evariste Galois was submitting his second draft of problem 5 as a student in this section of MTH 210, the attached file would be titled

Galois Problem 5 Draft 2.doc

In addition, you must use this title (without .doc) as your email subject line. So Evariste Galois would use the following in the email subject line:

Galois Problem 5 Draft 2

Important Guidelines and Rules for the Portfolio Project

1. You may not discuss the portfolio problems with anyone except the instructor of the course. This means that you are not allowed to talk to other students about your solutions, nor can you ask for help from the Math Lab tutors or other professors. Violation of this policy could result in a grade of zero on a given problem or possibly failure of the course. Please refer to the addendum of the syllabus dealing with plagiarism and academic honesty.
2. You may not use any sources to help complete the portfolio problems other than the textbook. This means that you are not allowed to use the Internet to find information about the problems unless specifically directed to do so as part of the problem. If you think you need some background information or a definition from another source, then ask me for permission. If permission is granted, then you may look up the necessary information and include it with a footnote in your proof.

3. All drafts that are submitted must be typeset and follow the guidelines for mathematical writing that are in the textbook.
4. You may hand in each of Portfolio Problems #1 through #11 to the professor two times for review. The third draft you submit for any of these Portfolio Problems will be graded and recorded as final.
5. No more than one Portfolio Problem may be submitted for review on a given day.
6. No more than four Portfolio Problems may be submitted for review during any given week. (A week will be considered to start on Monday.) However, no more than two portfolio problems may be submitted for review for the week beginning Monday April 7, 2008.
7. I will not discuss portfolio problems in my office after the posted deadlines (see below) for submitting drafts.

Deadlines for Submitting Portfolio Problems**Group 1 Portfolio Problems**

There are three problems in Group 1.

- The deadline for submitting Portfolio Problems 1 through 4 for review is Friday February 8, 2008.
- The last day to submit Portfolio Problems 1 through 4 for a grade is Friday February 15, 2008.

Group 2 Portfolio Problems

There are four problems in Group 2.

- The deadline for submitting Portfolio Problems 1 through 4 for review is Friday March 14, 2008.
- The last day to submit Portfolio Problems 1 through 4 for a grade is Friday March 21, 2008.

Group 3 Portfolio Problems

There are four problems in Group 3.

- The deadline for submitting Portfolio Problems 1 through 4 for review is Wednesday April 9, 2008.
- The last day to submit Portfolio Problems 1 through 4 for a grade is Wednesday April 16, 2008.

Final Portfolio Problem

This problem you choose is to be done without submitting it to the instructor for review. The deadline for submitting this problem for grading is Friday April 18, 2008.

Grading of the Portfolio Problems

Each problem in your portfolio will be graded on a 10-point scale with the only possible grades being 10, 9, 6, 3, or 0 points. There will be little partial credit because of the opportunity to submit problems for review, to re-write, and to re-submit. In order to receive full credit for a problem, your solution must be correct, complete, and well written with no spelling or grammatical errors. Following is a description of the 10-point scale for grading each problem:

Points	Description
10	The proof is mathematically correct and written according to the guidelines in the text plus those that follow.
9	The proof is mathematically correct but there is a minor error (note singular) in writing.
6	Significant mathematical progress has been made towards a proof but either the argument has one major error or the proof is not written according to the guidelines.
3	There is evidence of having some good ideas for constructing a proof and making an effort to write a formal proof.
0	Little or no progress has been made in developing a proof.

Grading of the Portfolio Project

The portfolio will be worth a total of 140 points. Each problem will be worth 10 points (for a total of 120 points), and in addition, there will be 20 points possible for submission of proofs for review by the professor. To be eligible for the 20 points, a student must do all of the following:

- Submit the first draft of a portfolio problem from Group 1 on or before Friday January 25, 2008.
- Submit the first draft of a second portfolio problem from Group 1 on or before Friday February 1, 2008.
- Submit the first draft of a portfolio problem from Group 2 on or before Friday February 22, 2008.
- Submit the first draft of a second portfolio problem from Group 2 on or before Friday February 29, 2008.
- Submit the first draft of a third portfolio problem from Group 2 on or before Wednesday March 12, 2008.
- Submit the first draft of a portfolio problem from Group 3 on or before Friday March 28, 2008.

If you meet these six deadlines, your score for this part of the portfolio will be 20 out of 20 points. For each deadline that is missed, 4 points will be deducted from your score but your score can be no lower than zero.

Honor System

All work that you submit for the Portfolio Project must be your own work. This means that you may not discuss the portfolio project with anyone except the instructor of the course and may not use any resources other than the textbook.

This will also provide me with information regarding how students are doing with each problem. So, if I find that a particular problem is causing more difficulties than anticipated, I can send an email message to all students with hints or points of clarification for that problem.

Some anticipated questions about this Portfolio

The answers to these questions contain some very important requirements and guidelines for the Portfolio Project.

What other requirements are there for my Portfolio Problems?

The solution for each problem must be written using complete sentences and according to the writing guidelines specified in the text. It must be neat, well organized, and easy to read. Proper grammar, proper sentence and paragraph structure, and correct spelling are necessities.

What happens if I submit an incorrect or incomplete solution?

The professor will return your problem and indicate if it is ready for your Portfolio or if it needs more work. When you submit a solution for a problem before the last day for review, you are asking the professor, Is this good enough for my Portfolio?

Should I wait and submit all my problems for review on the last day?

NO!! Begin working on your Proofs Project immediately. As soon as you have a proposed solution for a problem, you should write your solution and submit it for review. To encourage this:

- No more than one proof may be submitted for review on a given day (this includes February 8, March 14, and April 9).
- No more than four proofs may be submitted during any given week. A week is considered to start at 12:01 a.m. on Monday.
- No more than two portfolio problems may be submitted for review for the week beginning Monday April 7, 2008.

Can I work with someone else or use sources other than the textbook?

The only person you can discuss these problems with is the instructor for the course and the only resource you may use is the textbook. Plagiarism is not acceptable and will not be tolerated. No credit will be given for the solutions of problems in which plagiarism is involved.

What criteria will be used to judge my proofs?

A proof must be logically and mathematically correct. In addition, it must be written according to the course guidelines as developed in the text and discussed in class.

How should I start working on a particular problem?

Before beginning your proof or solution of the problem, you should make a clear statement of exactly what it is that is given in problem (the assumptions) and what is to be proven (the goal). That is, you should analyze the theorem or problem by carefully examining what is given or assumed and precisely what it is that will be proven. In this analysis, you should include any relevant definitions that are needed to clarify the statement of the problem. You should also elaborate on the assumptions made and the strategies that can be used to prove what it is that you are trying to prove. If it is appropriate, you must also include some examples to illustrate the problem. An example that illustrates this procedure is at the end of this document.

What are the writing guidelines for writing the solutions of the Portfolio Problems?

To receive full credit, the solution of a Portfolio Problem must be of collegiate quality and follow the writing guidelines for this course that are given in the textbook. This means that, in addition to demonstrating mastery of the subject matter, the solution should be neat and easy to read, well organized, and use proper grammar and spelling. In addition, a solution must meet the following guidelines:

- You should begin your presentation with a carefully worded statement of the problem. Do not use phrases such as “Show that” or “Prove that.” You should state the problem using simple declarative sentences. Following is a typical textbook problem.

Prove that if n is an integer and n^2 is odd, then n is odd.

If you were writing a solution to this problem for one of these portfolio problems, you should begin something like the following:

Theorem: If n is an integer and n^2 is an odd integer, then n is an odd integer.

- All calculations and algebraic manipulations must be clearly shown. By doing so, both you and your professor can follow the process you used to obtain an answer. Without a step-by-step presentation, it may be impossible to understand your solution, or if a mistake is made, it may be impossible to determine where a mistake was made.
- You might start your solution with a short discussion of the strategy that you will use. This is required if you use an indirect method of proof such as a proof by contradiction or the use of the contrapositive statement. In addition, you should conclude any proof with a statement of what has been proven, or minimally, that the proof is now complete.

An Example of a Well-done Portfolio Problem

This includes an analysis of the problem.

Problem X Is the following proposition true or false? Justify your conclusion.

If x and y are real numbers, then $\frac{x+y}{2} \geq \sqrt{xy}$.

If the proposition is true, write a complete proof for the proposition. If it is false, add a reasonable condition to the hypothesis so that the new proposition is true. Then, write a complete proof of this new proposition.

Portfolio Problem X

Proposition. If x and y are real numbers, then $\frac{x+y}{2} \geq \sqrt{xy}$.

This proposition is false as is shown by the following counterexample: If $x = -2$ and $y = -2$, then

$$\frac{x+y}{2} = -2 \text{ and } \sqrt{xy} = 2.$$

In this case, $\frac{x+y}{2} < \sqrt{xy}$. This shows that the given proposition is false.

Even though the proposition is false, we will explore the relationship between $\frac{x+y}{2}$ and \sqrt{xy} with specific examples.

x	y	$\frac{x+y}{2}$	\sqrt{xy}
2	2	2	2
4	1	$\frac{5}{2}$	2
-3	3	0	Not a real number
-3	-3	-3	3

Based on these examples (and several others), it appears that if x and y are positive real numbers, then $\frac{x+y}{2} \geq \sqrt{xy}$. We will state this as a theorem and prove it.

Theorem. If x and y are positive real numbers, then $\frac{x+y}{2} \geq \sqrt{xy}$.

Proof. We assume that x and y are positive real numbers and will show that $\frac{x+y}{2} \geq \sqrt{xy}$. Since the square of any real number is greater than or equal to zero, we know that $(x-y)^2 \geq 0$. Expanding the left side of this inequality gives

$$x^2 - 2xy + y^2 \geq 0.$$

We now add $4xy$ to both sides of this inequality. This is done so that the left side will become the square of $(x + y)$ as is shown below.

$$\begin{aligned}x^2 - 2xy + y^2 + 4xy &\geq 0 + 4xy \\x^2 + 2xy + y^2 &\geq 4xy \\(x + y)^2 &\geq 4xy\end{aligned}$$

We now take the square root of both sides of the last inequality. Since x and y are positive real numbers, $(x + y)$ is positive and hence, $\sqrt{(x + y)^2} = x + y$. In addition, $4xy$ is positive and so $\sqrt{4xy}$ is a real number. Since the square root function is an increasing function, the inequality is retained and we see that

$$\begin{aligned}\sqrt{(x + y)^2} &\geq \sqrt{4xy} \\(x + y) &\geq 2\sqrt{xy}\end{aligned}$$

If we now divide both sides of the last inequality by 2, we obtain $\frac{x + y}{2} \geq \sqrt{xy}$, which is what we were trying to prove. Hence, we have proven that if x and y are positive real numbers, then $\frac{x + y}{2} \geq \sqrt{xy}$. ■

Analysis of the Theorem (done before writing a proof)

This portion is not required and should not be submitted with your final solution. However, it is a good idea to do something like this to organize your work. It is often very useful to do this type of analysis before attempting to write a proof. Examples should help you understand the problem better.

Assumptions

The only assumption for this theorem is that x and y are positive real numbers.

Goal

The goal is to prove that $\frac{x + y}{2} \geq \sqrt{xy}$. For this, recall that a square root of a positive real number a is a real number b such that $a = b^2$. A square root can be positive or negative but the symbol \sqrt{a} represents the positive square root of a .

One way to find a method to prove this is to “work backwards” from the goal. If we square both sides of the inequality that is the goal, we obtain

$$\begin{aligned}\frac{(x + y)^2}{4} &\geq xy \\x^2 + 2xy + y^2 &\geq 4xy \\x^2 - 2xy + y^2 &\geq 0 \\(x - y)^2 &\geq 0\end{aligned}$$

This, of course, does not constitute a proof of the theorem since we started with “the goal.” However, we may be able to reverse these steps to construct a proof.