Guidelines for The Portfolio Project

Description of the Portfolio Project

During the semester, ten problems will be posed to all students. Each student will work on these problems and submit proposed solutions to the professor at the end of the semester. In addition, each student will submit a completed list of definitions for the portfolio. You may bring each of your portfolio problems and the list of definitions to the professor two times to be critiqued. The professor will make recommendations about these and give a "temporary grade" for the problem. You then have the opportunity to rewrite and resubmit your drafts for further comment.

Important guidelines and rules for the Portfolio Project

- You may not discuss the portfolio problems with anyone except the instructor of the course.
- You may not use any sources to help complete the portfolio problems other than the textbook.
- You may hand in a given Portfolio Problem to the professor two times to be critiqued.
- No more than one Portfolio Problem may be submitted for review on a given day.
- No more than four Portfolio Problems may be submitted for review during any given week. (A week will be considered to start on Monday.) However, no more than two portfolio problems may be submitted for review for the week beginning Monday April 7, 2003.
- The deadline for having a Portfolio Problem critiqued is the beginning of class (10:00 a.m.) on Wednesday April 9, 2003.
- The final Portfolio is due no later than the beginning of class (10:00 a.m.) on Wednesday April 16, 2003.

Grading of the Portfolio Project

The portfolio will be worth a total of 150 points. Each problem will be worth 10 points (for a total of 100 points), and the list of definitions will be worth 20 points. In addition, there will be 30 points possible for submission of proofs for review by the professor. To be eligible for the 30 points, a student must do all of the following:

- Submit the first draft of a portfolio problems by Friday January 31, 2003;
- Submit the first draft of a second portfolio problem (different than the first) by Wednesday February 12, 2003 and
- Submit the first draft of a third portfolio problem (different than the first two) by Wednesday February 19, 2003;
- Submit the first draft of the list of definitions through Part Three by Friday February 28, 2003;
- Submit the first draft of a fourth portfolio problem (different than the first three) by Friday March 14, 2003;
- Submit the first draft of a fifth portfolio problem (different than the first four) by Friday March 21, 2003; and
- Submit the first draft of a sixth portfolio problem (different than the first five) by Friday March 28, 2003;

If you meet these seven deadlines, your score for this part of the portfolio will be 30 out of 30 points. For each deadline that is missed, 5 points will be deducted from your score.

In addition, there will be ten "extra credit points" available for the Portfolio Project. These extra credit points will be awarded to each student who has:

- Received a score of 10 on three portfolio problems by Friday March 28, 2003; or
- Received a score of 9 or more on six portfolio problems by Friday March 28, 2003.

Honor System
All work that you submit for the Portfolio Project must be your own work. This means that you may not discuss the portfolio project with anyone except the instructor of the course and may not use any resources other than the textbook.

This will also provide me with information regarding how students are doing with each problem. So, if I find that a particular problem is causing more difficulties that anticipated, I can send a email message to all students with hints or points of clarification for that problem.

Electronic Submission of Portfolio Problems
Each solution or proof must be done on a word processor capable of producing the appropriate mathematical symbols and equations. Microsoft Word and its Equation Editor, which is available on the student network, is one such word processor.

Each solution or proof for a portfolio problem must be submitted to the instructor electronically through the Digital Drop Box that is on the course web page (through Grand Valley's Blackboard system). The instructor will make comments on the problem and return them to the student using the Digital Drop Box.

Following are some (anticipated) questions about this Portfolio. The answers to these questions contain some very important requirements and guidelines for the Portfolio Project.

What other requirements are there for my Portfolio Problems?
The solution for each problem must be written using complete sentences and according to the writing guidelines specified in the text. It must be neat, well organized, and easy to read. Proper grammar, proper sentence and paragraph structure, and correct spelling are necessities.

What happens if I submit an incorrect or incomplete solution?
The professor will return your problem and indicate if it is ready for your Portfolio or if it needs more work. When you submit a solution for a problem before the last day for review, you are asking the professor, "Is this good enough for my Portfolio?" Only the problems and definitions turned in on Wednesday April 16, 2003 will determine your score on those two portions of the portfolio.

When can I submit a proposed solution for a problem?
You may submit a problem for review any time before Wednesday April 9, 2003**.**

Should I wait and submit all my problems for review on the last day?
NO!! As soon as you have a proposed solution for a problem, you should write your solution and submit it for review. To encourage this, no more than one proof may be submitted for review on a given day (this includes April 9), and no more than four proofs may be submitted during any given week. (No more than two portfolio problems may be submitted for review for the week beginning Monday April 7, 2003.) **Begin working on your Proofs Project immediately.**

Can I work with someone else or sources other than the textbook?
The only person you can discuss these problems with is the instructor for the course and the only resource you may use is the textbook. ***Plagiarism is not acceptable*** and will not be tolerated. No credit will be given for the solutions of problems in which plagiarism is involved.

What criteria will be used to judge my proofs?
A proof must be logically and mathematically correct. In addition, it must be written according to the course guidelines as developed in the text and discussed in class.

How will my grade for a given problem be determined?
Each problem in your portfolio will be graded on a 10-point scale with the only possible grades being 10, 9, 6, 3, or 0 points. There will be little partial credit because of the opportunity to submit problems for review, to re-write, and to re-submit. In order to receive full credit for a problem, your solution must be correct, complete, and well written with no spelling or grammatical errors. Following is a description of the 10-point scale for grading each problem:

Points	Description
10	The proof is correct and written according to the guidelines in the text plus those that follow.
9	The proof is correct but there is a writing mistake.
6	The proof is essentially correct but the solution is not written according to the guidelines.
3	Significant progress has been made in developing and writing a proof for the theorem.
0	Little or no progress has been made in developing a proof for the theorem.

How should I start working on a particular problem?
Before beginning your proof or solution of the problem, you should make a clear statement of exactly what it is that is given in problem (the assumptions) and what is to be proven (the goal). That is, you should analyze the theorem or problem by carefully examining what is given or assumed and precisely what it is that will be proven. In this analysis, you should include any relevant definitions that are needed to clarify the statement of the problem. You should also elaborate on the assumptions made and the strategies that can be used to prove what it is that you are trying to prove. If it is appropriate, you must also include some examples to illustrate the problem. An example that illustrates this procedure is at the end of this document.

What are the writing guidelines for writing the solutions of the Portfolio Problems?

To receive full credit, the solution of a Portfolio Problem must be of collegiate quality and follow the writing guidelines for this course that are given in the textbook. This means that, in addition to demonstrating mastery of the subject matter, the solution should be neat and easy to read, well organized, and use proper grammar and spelling. In addition, a solution must meet the following guidelines:

- You should begin your presentation with a carefully worded statement of the problem. Do not use phrases such as "Show that" or "Prove that". You should state the problem using simple declarative sentences. Following is a typical textbook problem.

  Prove that if n is an integer and n^2 is odd, then n is odd.

  If you were writing a solution to this problem for one of these writing assignments, you should begin in the following manner:

  Theorem: If n is an integer and n^2 is an odd integer, then n is an odd integer.

- All calculations and algebraic manipulations must be clearly shown. By doing so, both you and your professor can follow the process you used to obtain an answer. Without a step-by-step presentation, it may be impossible to understand your solution, or if a mistake is made, it may be impossible to determine where a mistake was made.

- You might start your solution with a short discussion of the strategy that you will use. This is required if you use an indirect method of proof such as a proof by contradiction or the use of the contrapositive statement. In addition, you should conclude any proof with a statement of what has been proven, or minimally, that the proof is now complete.

Following is an example of a well-done Portfolio Problem, which includes an analysis of the problem.

Problem X

Is the following proposition true or false? Justify your conclusion.

If x and y are real numbers, then $\frac{x+y}{2} \geq \sqrt{xy}$.

If the proposition is true, write a complete proof for the proposition. If it is false, add a reasonable condition to the hypothesis so that the new proposition is true. Then, write a complete proof of this new proposition.

Portfolio Problem X

Proposition

If x and y are real numbers, then $\frac{x+y}{2} \geq \sqrt{xy}$.

This proposition is false as is shown by the following counterexample: If $x = -2$ and $y = -2$, then

$$\frac{x+y}{2} = -2 \text{ and } \sqrt{xy} = 2.$$

In this case, $\frac{x+y}{2} < \sqrt{xy}$. This shows that the given proposition is false. ■

We will now compute the quantities $\frac{x+y}{2}$ and $\sqrt{xy}$ for various values of x and y. The results are shown in the following table:

x	y	$\frac{x+y}{2}$	$\sqrt{xy}$
2	2	2	2
4	1	$\frac{5}{2}$	2
-3	3	0	Not a real number
-3	-3	-3	3

Based on these examples (and others), it appears that if x and y are positive real numbers, then $\frac{x+y}{2} \geq \sqrt{xy}$. We will state this as a theorem and prove it.

Theorem: If x and y are positive real numbers, then $\frac{x+y}{2} \geq \sqrt{xy}$.

Proof: We assume that x and y are positive real numbers. The goal is to show that $\frac{x+y}{2} \geq \sqrt{xy}$. Since the square of any real number is greater than or equal to zero, we know that $(x-y)^2 \geq 0$. Expanding the left side of this inequality gives

$$x^2 - 2xy + y^2 \geq 0.$$

We now add $4xy$ to both sides of this inequality. This is done so that the left side will become the square of $(x+y)$ as is shown below.

$$\begin{aligned} x^2 - 2xy + y^2 + 4xy &\geq 4xy \\ x^2 + 2xy + y^2 &\geq 4xy \\ (x+y)^2 &\geq 4xy . \end{aligned}$$

We now take the square root of both sides of the last inequality. Since x and y are positive real numbers, $(x+y)$ is positive and hence, $\sqrt{(x+y)^2} = (x+y)$. So,

$$\begin{aligned} \sqrt{(x+y)^2} &\geq \sqrt{4xy} \\ (x+y) &\geq 2\sqrt{xy} . \end{aligned}$$

If we now divide both sides of the last inequality by 2, we obtain $\frac{x+y}{2} \geq \sqrt{xy}$, which is what we were trying to prove. Hence, we have proven that if x and y are positive real numbers, then $\frac{x+y}{2} \geq \sqrt{xy}$. ■

Analysis of the Theorem (done before writing a proof):
This portion is not required and should not be submitted with your final solution. However, it is a good idea to do something like this to organize your work. It is often very useful to do this type of analysis before attempting to write a proof. Examples should help you understand the problem better.

Assumptions:
The only assumption for this theorem is that x and y is positive real numbers.

Goal:
The goal is to prove that $\frac{x+y}{2} \geq \sqrt{xy}$. For this, recall that a square root of a positive real number a is a real number b such that $b^2 = a$. A square root can be positive or negative but the symbol $\sqrt{a}$ represents the positive square root of a.

One way to find a method to prove this is to "work backwards" from the goal. If we square both sides of the inequality that is the goal, we obtain

$$\frac{(x+y)^2}{4} \geq xy$$
$$x^2 + 2xy + y^2 \geq 4xy$$
$$x^2 - 2xy + y^2 \geq 0$$
$$(x-y)^2 \geq 0 \quad .$$

This, of course, does not constitute a proof of the theorem since we started with "the goal." However, we may be able to reverse these steps to construct a proof.

Examples:

If $x = 4$ and $y = 9$, then $\frac{x+y}{2} = \frac{13}{2}$ and $\sqrt{xy} = \sqrt{36} = 6$. In this case, $\frac{x+y}{2} \geq \sqrt{xy}$.

If $x = 12$ and $y = 20$, then $\frac{x+y}{2} = \frac{32}{2} = 16$ and $\sqrt{xy} = \sqrt{12 \cdot 20} = \sqrt{240}$. In this case, $\frac{x+y}{2} \geq \sqrt{xy}$.