Team Project and Presentation

Each student will work as a member of a team of 3 students on this project - I will solicit your choices for team members, but will make the decisions regarding who works with whom. This project will involve the selection of an important topic/person in the development of modern mathematics, researching the nature, history, and implications of the idea, and then writing a substantial paper on the topic. In addition, each team will make a 15 minute presentation to the class about their topic and their research of the topic. These presentations will be done during the last week of class. The entire project will be 25% of your course grade.

The semester project will be to write a chapter of a book - envision a book like *Journey through Genius* or *Euler, the Master of Us All*, but instead on another mathematician (or mathematicians) or great theorem. Your paper will include a prologue (a history of the subject up until your mathematician(s) enters or the theorem was proven), then at least one major contribution of that (those) mathematician(s) or of the significance of the theorem, discussed in full detail. The paper will conclude with an epilogue, an examination of how the mathematics in this area has developed since your mathematician(s) or the theorem was proven.

This project first involves picking an appropriate historical and mathematical subject, and if appropriate, at least one mathematician of significance in the area. The topic should be interesting (both to you and a wider audience), and needs to include a substantial proof (or a significant mathematical idea) that you will explain to an audience that is not as mathematically sophisticated as you are - envision writing to a group of students who know some calculus and have completed math 210. Your research paper will be 12 to 15 pages long, complete with references.

The point breakdown for the project is included in the discussion of dates below.

1. **By Tuesday September 13**: Submit the names of three potential classmates with whom you wish to work (it is preferable that you speak with each person first), the topic(s) you are interested in pursuing, and at least four one to two hour time blocks in which you are free (every week) to meet to discuss the project.

   From this information, I will attempt to pair project teams of people with suitably agreeable schedules and interests. Only one group can work on a given topic. Teams will be announced by Tuesday September 20.

2. **By Tuesday October 4**: Turn in a typeset summary of the historical context of your problem, topic, or mathematician, including all relevant names and dates. This is essentially the first portion of the paper - the Prologue. This introduction should be 3-4 pages long. Include all references used, and cite them appropriately in your writing. (10 points)

3. **By Tuessday October 25**: Turn in a typeset outline of your entire paper including plans for the epilogue. Included will be a short peer evaluation of your co-worker on the project. (10 points)

4. **By Thurssday November 10**: Turn in a rough draft of your full paper. This will be typeset and double-spaced. Your report should be 12 to 15 pages long and must adhere to the standard writing guidelines from Writing 150 and MTH 210. (10 points)

   At least a week prior to this date you will get a copy of the grading guidelines I will use on the paper. This draft will be returned to you with my comments by Thursday November 17.

5. **By Tuesday December 6**: Turn in the final draft of your paper to me. Papers will be graded and returned by our final examination. Classroom presentations will be made on December 6 and December 8.

   The final paper will be marked on a scale of 80 points, and the presentation will be graded on a scale of 40 points.

6. **On Thursday December 8**: After class, turn in your peer evaluations of the other members of your team, along with the self-evaluation of your own work. The self, peer, and instructor evaluations of your contributions to the project will be used to determine any necessary adjustments to the grade each individual will receive.

Topics for the Project

Now that you know the basics of the project, what should you learn and write about? A list of suggested topics can be found below. You would not necessarily need to answer every question under the suggested topic; those are just to give you something to think about. Feel free to choose a topic not on this list, but remember that you want a result that can be communicated to a large audience, and the instructor must approve topics not on the list.

1. Prime number conjectures and open questions. There are several fascinating open problems dealing with prime numbers, such as: (i) Goldbach's conjecture; (ii) Are there infinitely many primes of the form $n^2 + 1$ where n is a natural number?; (iii) Is there always a prime number between n^2 and $(n+1)^2$? Report on at least one open problem and progress that has been made in resolving the problem.

2. Public key encryption systems. (If you did a project in any other course dealing with the RSA encryption system, you must choose another topic.) What is a public key encryption system? How are they different than other methods of cryptography? Why are they important? What does RSA stand for? Why does the RSA system work?

3. The Prime Number Theorem. What does the Prime Number Theorem tell us? Are mathematicians interested in other theorems that tell us about the distribution of certain types of primes? Why? Who originally proved the Prime Number Theorem? Have there been "better" proofs since?

4. The Mandelbrot set. What is this famous set? How is it constructed? What are some interesting results that have been proven about this mathematical object?

5. Who were Galois and Abel? What were their contributions to mathematics? Why is their work considered to be the start of "modern algebra?" What does it mean to say that a polynomial equation is "solvable by radicals?" What mathematicians worked on their main problems prior to them? (This is briefly discussed in the epilogue of Chapter 6 in *Journey through Genius*. Some familiarity with group theory may be helpful.)

6. Fermat's Last Theorem. We may see a video about this theorem in class. Your report, however, should be more thorough and deal with some of the history of the theorem and attempts to prove it. You will not be able to include a complete proof of this theorem in your report, but you should be able to include proofs of some special cases.

7. The construction of the real numbers from the rational number system. In this course, we have seen how to construct the integers from the natural numbers. In MTH 310, we showed how to construct the rational numbers from the integers as the field of quotients of the integers. So, if we start with the integers, then we know what the rational numbers are? But what are the real numbers? To construct the real numbers, you will need to use Dedekind cuts or Cauchy sequences.

8. The history of angle trisections. Toward the end of this course, we will probably prove that it is impossible to devise a method using only compass and straightedge that will trisect any angle. Your report should discuss the history of the angle trisection problems. It should include some proposed methods that did not work, including a discussion of why they did not work, and it should include some angle trisection methods that use tools other than the compass and straightedge.

9. The quaternions. If we consider the complex numbers as a two dimensional space over the real numbers, is it possible to construct a three dimensional algebra over the real numbers? The quaternions were introduced by William Rowan Hamilton of Ireland in 1843. Hamilton was looking for ways of extending complex numbers (which can be viewed as points on a plane) to higher spatial dimensions. What are the quaternions? Why was the use of quaternions considered controversial in the nineteenth century? Is the use of the quaternions still controversial? What are some applications of the quaternions?

10. The mathematical theory of apportionment. This could be a good project if you are interested in the history of the United States, and in particular, the history of representation methods for the House of Representatives. What are some of the famous apportionment methods that have been used? How is "fairness" defined for an apportionment method? What does it mean to say that an apportionment method is "unbiased?" Who are Balinski and Young and why is their theorem about apportionment methods important?

11. Arrow's theorem. (You cannot do this if you are currently enrolled in MTH 330.) Is it possible to have an election method that satisfies a fairly modest set of "fairness criteria?" What are some of the standard election methods? This theorem is named after economist Kenneth Arrow, who proved the theorem in his Ph.D. thesis and popularized it in his 1951 book *Social Choice and Individual Values*. Arrow was a co-recipient of the 1972 Nobel Prize in Economics.

12. Modern geometry. What are the theorems of Ceva, Menelaus, and Morley? How are they proved? Are there other surprising results that have arisen in the last 100 years or so that eluded the Greeks?

13. The Fundamental Theorem of Arithmetic: What does this theorem say? Who first proved it? Can you prove it? What is the significance of this theorem in how it is used in mathematics today? Include at least two key applications of this theorem.

14. The Fundamental Theorem of Algebra: What does this theorem say? Who first conjectured it? Who first proved it? What is the basic argument behind the proof? (Note: some experience with complex variables is helpful here.) How is the Fundamental Theorem of Algebra related to the work of mathematicians like Galois and Abel?

15. The number e is irrational (indeed transcendental): What is the history of e? Who was the first to prove e is irrational? What is a transcendental number? Can we prove that e is transcendental? Are there other famous transcendental numbers?

16. Famous problems of geometry. With only a ruler and a compass, is it possible to trisect any angle? Is it possible to double the cube? Is it possible to square the circle? How was algebra used to answer these famous Greek geometric questions? What are constructible numbers and how were they used to help answer these questions? Are there methods that can be used to trisect any angle?

17. The Binomial Theorem: Who first proved the Binomial Theorem? Why is it so important? What other mathematics relies heavily on the result? Is there a best proof of this theorem?

18. A biography and “mathematical biography” of a famous mathematician.