Analysis of Type 1 Diabetes with Wavelets and Other Mathematical Tools

GVSU REU 2009

Edward Aboufadel
Department of Mathematics
Grand Valley State University
Allendale, MI 49401

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Presentation Overview

1. Introduction Type 1 Diabetes

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2. Introduction to Wavelets

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3. Project 1: Clustering Diabetics

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4. Project 2: Glucose Control with a Wavelet-Based Controller

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1. Introduction Type 1 Diabetes
2. Introduction to Wavelets
3. Project 1: Clustering Diabetics
4. Project 2: Glucose Control with a Wavelet-Based Controller
5. How We'll Get Started

Type 1 Diabetes

Zachary Aboufadel: hockey player, 5th grader, type 1 diabetic
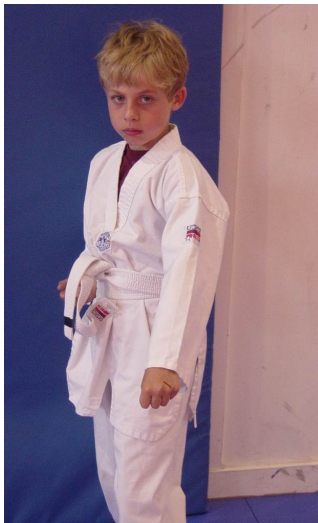
Type 1 Diabetes

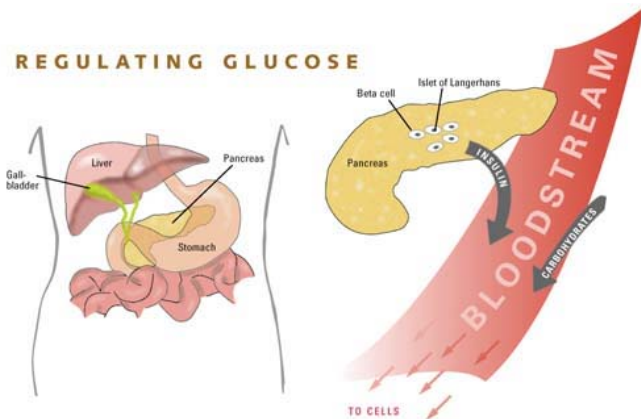Zachary Aboufadel: hockey player, 5th grader, type 1 diabetic

Type 1 Diabetes

- Beta cells in the pancreas create insulin to regulate the amount of glucose in the blood.
- Insulin is a hormone that helps the body move the glucose contained in food into cells throughout the body, which use it for energy.
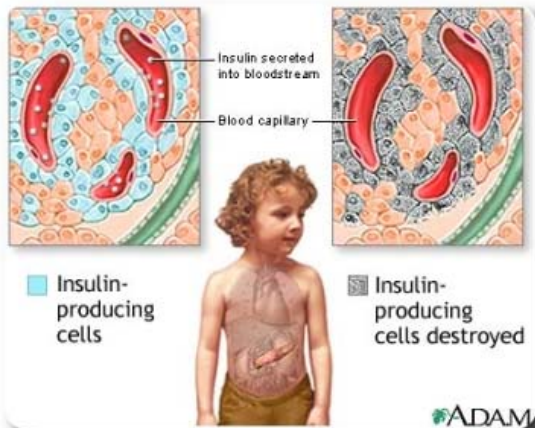
Type 1 Diabetes

Type 1 Diabetes

- For people with type 1 diabetes, the beta cells have been destroyed.
- When the beta cells are destroyed, no insulin can be produced, and the glucose stays in the blood instead, where it can cause serious damage to all the organ systems of the body.
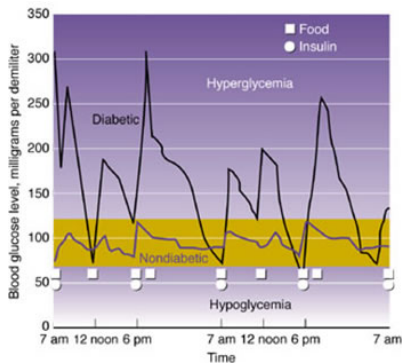
Type 1 Diabetes

Type 1 Diabetes

- The beta cells are destroyed by the body's own immune system, so type 1 diabetes is considered an autoimmune disease.
- Something happens in the body to activate the immune system to attack the beta cells, but medical scientists are not certain what that is.

- There is growing but inconclusive evidence that type 1 diabetes starts with an infection by a type of virus called a coxsackie virus, which are polio-related viruses that cause upper respiratory infections. After fighting off the virus, the body's immune system turns on the beta cells in the pancreas.
- However, you cannot catch diabetes from a type 1 diabetic.

Managing Type 1 Diabetes

- For someone with type 1 diabetes, the most important job is keeping the blood glucose level under control.
- Non-diabetics typically have a blood glucose reading between 80 and 100 mg per dL.
- For diabetics, that number can range from 20 to over 500.

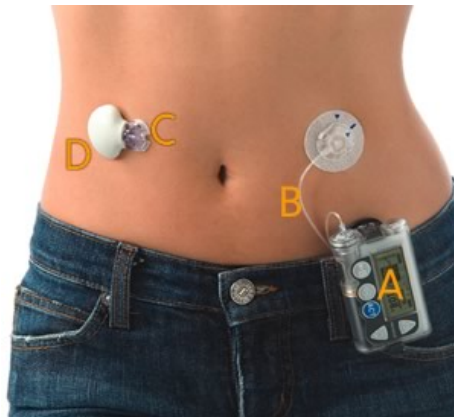# Managing Type 1 Diabetes

Diabetic vs. Non-Diabetic

Managing Type 1 Diabetes

- Many type 1 diabetics use an insulin pump to put insulin into their bloodstream 24 hours a day.

- For research purposes, a continuous glucose meter can be used to record glucose levels at 5-minute intervals.

- For most people, though, a meter with test strips is used four to eight times a day.

Managing Type 1 Diabetes
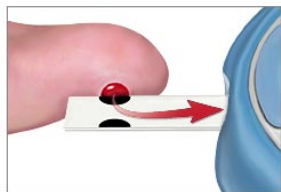
Insulin Pump and Continuous Glucose Monitor

Managing Type 1 Diabetes

Glucose Meter

Managing Type 1 Diabetes

Testing With A Meter

Managing Type 1 Diabetes

- About 24 million Americans have diabetes now, with 5-10% being type 1 diabetes. There are about 30,000 new cases of type 1 diagnosed each year.

- Recent studies suggest that the prevalence of the disease is increasing.

- Some well-known type 1 diabetics: Chicago Bears QB Jay Cutler; Nick Jonas (of the Jonas brothers); actress Sharon Stone; American Idol contestant Elliot Yamin; actress Mary Tyler Moore; musician David Crosby; supreme-court nominee Sonia Sotomayor.

Managing Type 1 Diabetes

Diabetic vs. Non-Diabetic
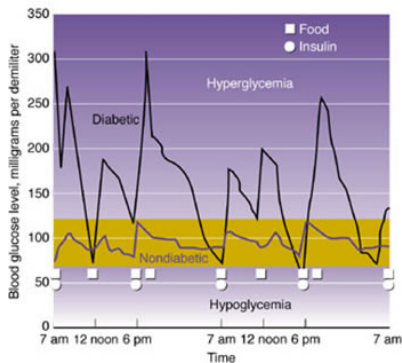
Wavelets

Wavelets have been developed over the past 30 years. They are a tool that have been used for image compression, denoising old sound recordings, and many other applications.

Functions or Filters

- You can think of wavelets as functions that are used to analyze other functions.
- You can think of wavelets as filters that are used to analyze signals.

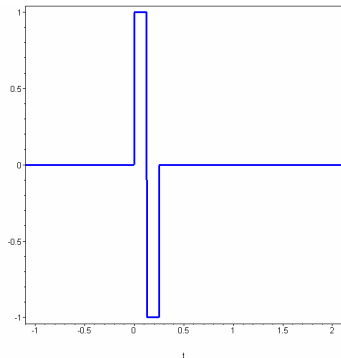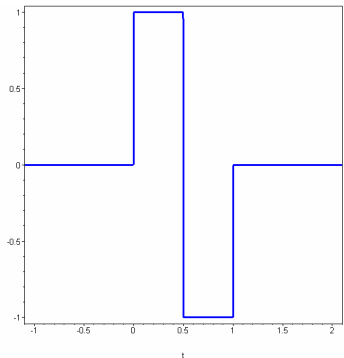# Wavelets to Analyze Functions
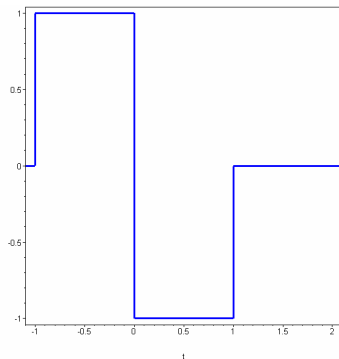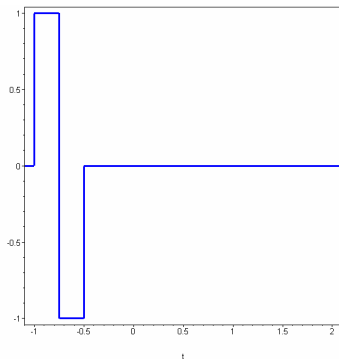
The Haar mother wavelet
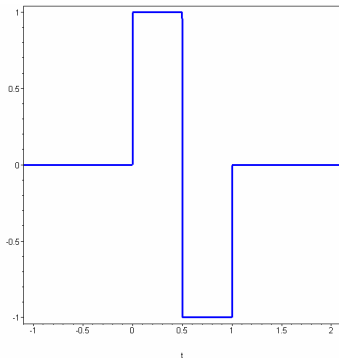
Wavelets to Analyze Functions

Shrinking the Haar mother wavelet

Wavelets to Analyze Functions

Dilations and Translations of the Haar mother wavelet

Wavelets to Analyze Functions

Wavelet Multiresolution Analysis of a Function

$\psi(t)$ is the mother wavelet. $f(t)$ is the function to be decomposed.

$$f(t) \approx \sum_{j,k} a_{j,k} \psi(2^j t - k) \tag{1}$$

Wavelets to Analyze Functions

- In a wavelet multiresolution analysis, a set of wavelets forms a basis for a vector space of functions.
- Other functions are written as a linear combination of the wavelet basis functions.
- The coefficients in the linear combination can tell you something about the function you are analyzing, or can be used for other purposes.

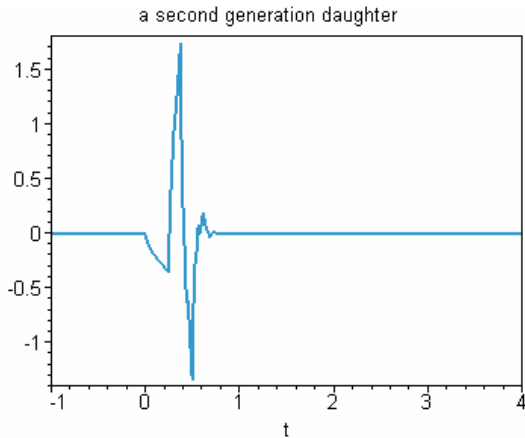# Wavelets to Analyze Functions

The Daubechies father wavelet
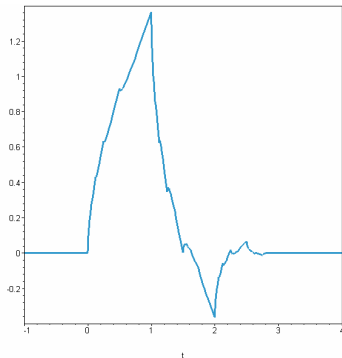
Wavelets to Analyze Functions
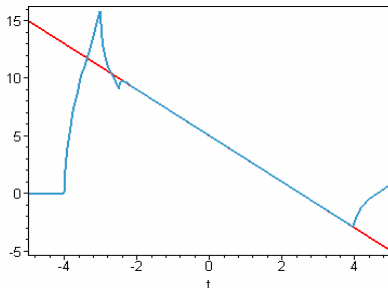
A Daubechies daughter wavelet

a second generation daughter

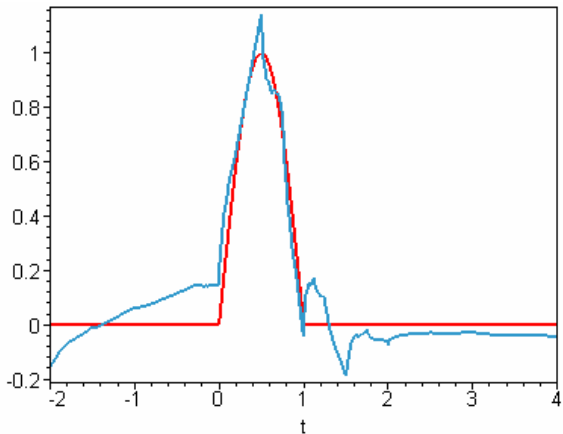Exact reproduction of a linear function

$$\sum_{k=-4}^{3} (-2k + 2 + \sqrt{3})D_4(t-k)$$

Wavelets to Analyze Functions

Linear Combination of 15 Daubechies wavelet functions

Wavelets to Analyze Signals

- The simplest signal is just a sequence of numbers.
- A wavelet filter converts a signal into another signal. The output can tell you something about the original signal, or can be used for other purposes.

Wavelets to Analyze Signals

Example with Daubechies Filters

- Here is the signal we are going to analyze: $s =$ [1.02, 2.01, 2.99, 4.05, 5.12, 6.03, 7.09, 7.98, 9.14, 10.05, 11.02, 12.00].
- Here are the filters:
- Low-pass filter: $H = [0.483, 0.837, 0.224, -0.129]$.
- High-pass filter: $G = [-0.129, -0.224, 0.837, -0.483]$.
- When filtering, we basically compute the dot product between four entries in the signal and the four entries in the filter.
- The low-pass filter will compute some sort of average of the signal entries. The high-pass filter will pick up changes or differences in the signal.

Wavelets to Analyze Signals

Example with Daubechies Filters

- $s =$ [1.02, 2.01, 2.99, 4.05, 5.12, 6.03, 7.09, 7.98, 9.14, 10.05, 11.02, 12.00].
- Low-pass results: $Hs =$ [2.32234, 5.20303, 8.07881, 10.85464, 13.74695].
- High-pass results: $Gs =$ [–0.03534, 0.08004, 0.06879, 0.09390, –0.00252].
- The high-pass results are close to zero because the signal is practically linear.
- Filtering can be reversed, but you can tweak the output before reversing as a way to clean up data, or hide messages, or something else.
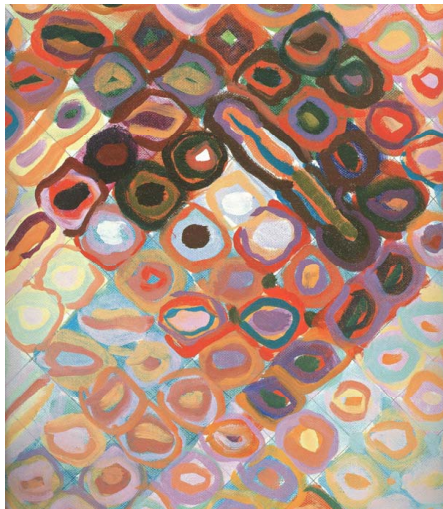
Artwork by Chuck Close

REU 2008: Imitating Chuck Close
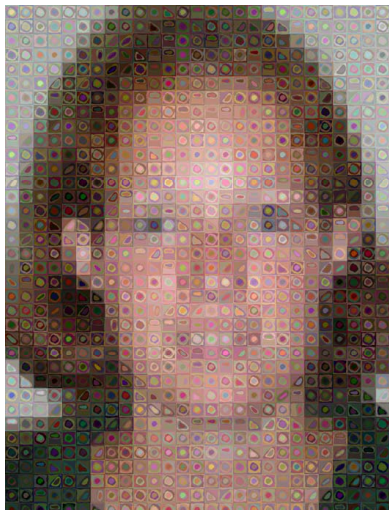
Artwork by Chuck Close

by Clara Madsen and Sarah Boyenger
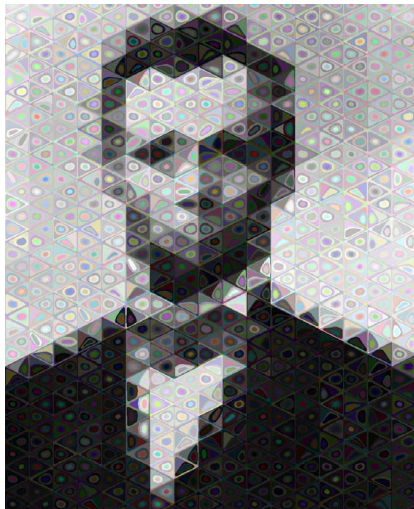
Something More Abstract

REU 2009: Project 1 – Clustering Diabetics

- The AIDA Online Diabetes Simulator can generate blood glucose time sequences for many sample cases.
- If we think of each sequence as a node in a graph, can we develop a way to determine which sequences are adjacent, using wavelets? (I have a few ideas, and perhaps you will, too.)
- If so, we can use a new method of data analysis called the Laplacian eigenmap method to cluster the diabetics.
- The reason to do this: diabetics in a cluster might be managed in a certain way, while diabetics in another cluster might need a different type of treatment.

REU 2009: Project 1 – Clustering Diabetics

- I am about to get access to real data (time sequences) from 137 diabetics who wore continuous glucose monitors for a full year.
- We can apply our wavelet-based clustering methods to this data to see what we can determine.

REU 2009: Project 2 – A Wavelet-Based Controller for an Artificial Pancreas

- In industry, PID controllers are used to minimize errors in manufacturing.
- PID = Proportional, Integral, Derivative.
- P = the immediate error.
- I = the accumulated error.
- D = the immediate change in the error.

REU 2009: Project 2 – A Wavelet-Based Controller for an Artificial Pancreas

- One idea that is being used in research for an artificial pancreas is to have it work like a PID controller.
- There is some evidence that a normal pancreas really works that way.
- I have a paper from 2005 proposing a wavelet-based PID controller.
- Is it a good proposal? Can we do better? How could we apply this to glucose control?

REU 2009: Getting Started

- A lot of reading to do this week: Diabetes, Wavelets, Eigenmap.
- We should be able to get the eigenmap method up-and-running without too much trouble.
- At least one of the team members needs to have some experience with programming.
- We can't go in to this thinking we'll really make a difference, but it will be fun to try.