ACCELERATING REFERENCE FRAMES: ROTATING FRAMES

I. Review of fictitious "forces"

The following fictitious "forces" are used to predict and account for the motion of objects in non-inertial reference frames:

Inertial "force": $\vec{F}'_{inertial} = -m\vec{A}$

Coreolis "force": $\vec{F}'_{Coreolis} = -2m\vec{\omega}\times\dot{\vec{r}}'$

Centrifugal "force": $\vec{F}'_{centrifugal} = -m\vec{\omega}\times\left(\vec{\omega}\times\vec{r}'\right)$

Transverse "force": $\vec{F}'_{transverse} = -m\dot{\vec{\omega}}\times\vec{r}'$

(*Note:* The primed quantities describe the motion of an object with respect to a non-inertial frame, while $\vec{A}$, $\vec{\omega}$, and $\dot{\vec{\omega}}$ describe the motion of the non-inertial frame with respect to an inertial frame.)

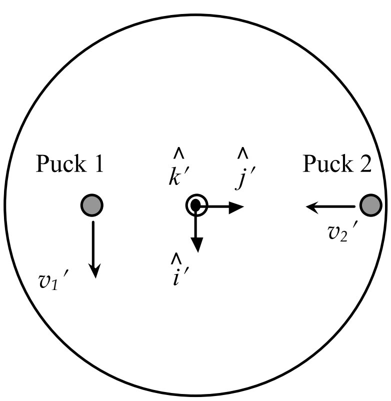
For each of the fictitious "forces" above, briefly describe *in words* how a non-inertial frame must move relative to an inertial frame in order for that "force" to be considered important.

II. Motion on a rotating platform

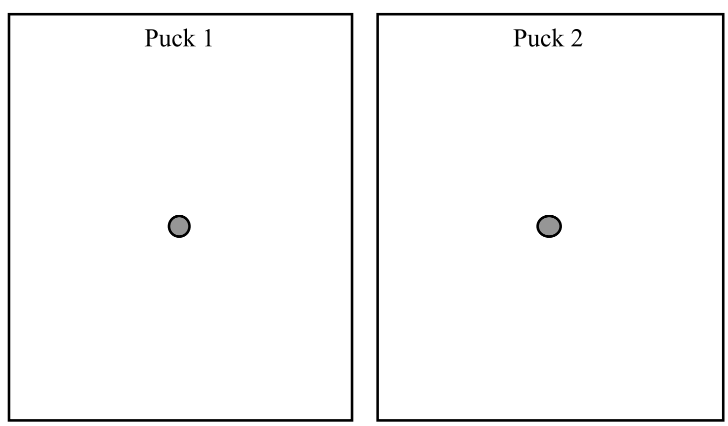
A level, frictionless platform rotates *counter-clockwise* at a constant rate relative to an inertial (or, "lab") frame. Consider a reference frame whose origin is at the center of the platform and that is stationary relative to the platform.

Two pucks (1 and 2) on the platform have initial velocities $\vec{v}_1'$ and $\vec{v}_2'$ *as measured in the platform frame.*

Top view diagram ***in platform frame***
(Platform spins CCW in lab frame)

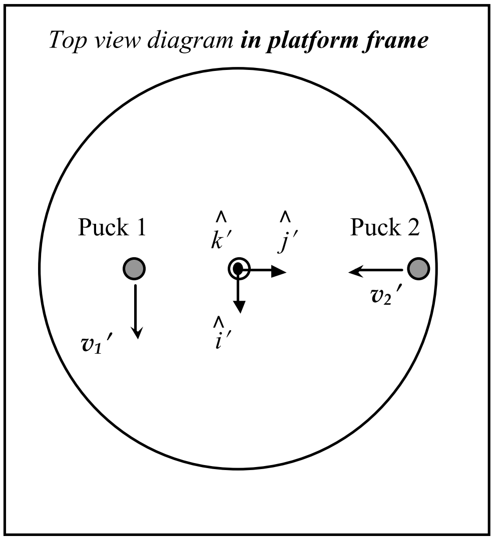


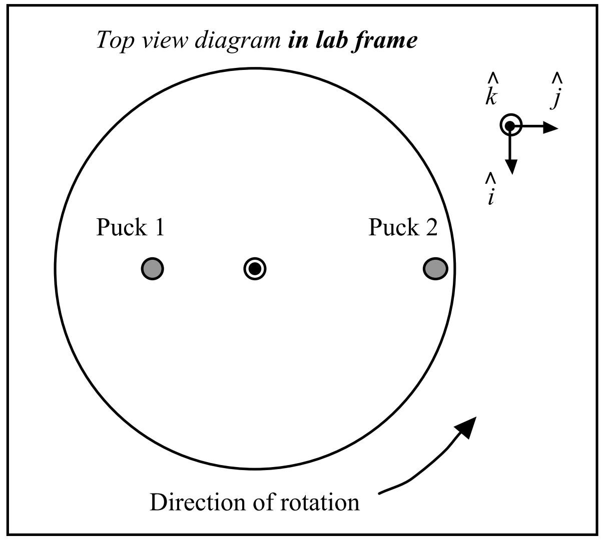
Free-body diagrams *in platform frame*



A. For the instant shown in the top view diagram, draw separate free-body diagrams for the two pucks, including all non-zero fictitious "forces." (Draw each diagram from a top view, *i.e.,* ignore the gravitational and normal forces on each puck.) Discuss your reasoning with your partners.

The top view diagrams below show the locations of the pucks at the same instant. The first diagram is drawn in the *platform* frame; the second, in the *lab* frame.





B. Would puck 2—initially moving in the $-\hat{j}'$ direction in the platform frame—reach the exact center of the platform? Use your free-body diagram for puck 2 to support your answer.

On the first diagram above, sketch a qualitatively correct path for each puck as viewed in the *platform frame.* Discuss your reasoning with your partners.

C. The second diagram above, drawn in the lab frame, represents the same instant shown in the first diagram. At that instant the primed unit vectors coincide exactly with the unprimed unit vectors.

1. Would puck 2—initially moving in the $-\hat{j}'$ direction in the *platform frame*—have an initial velocity *purely* in the $-\hat{j}$ direction in the *lab frame?* Explain why or why not.

   [*Hint:* Can puck 2 reach the exact center of the platform in one frame but not in the other?]

2. On the second diagram, sketch vectors $\vec{v}_1$ and $\vec{v}_2$ representing the initial velocities of the pucks as measured in the *lab* frame. Clearly show how the vectors $\vec{v}_1$ and $\vec{v}_2$ differ in magnitude and direction from the corresponding velocity vectors $\vec{v}_1'$ and $\vec{v}_2'$ for the pucks in the platform frame.

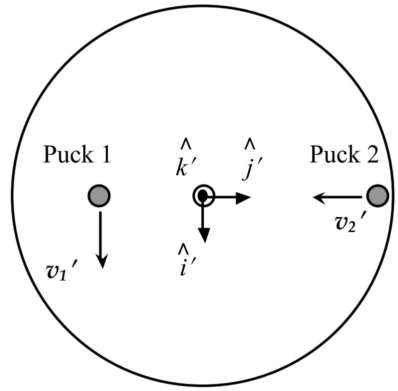
   Are your velocity vectors consistent with the relationship $\vec{v} = \vec{v}' + (\vec{\omega} \times \vec{r}')$? If not, resolve the inconsistencies.

3. On the basis of your results, sketch a qualitatively correct path for each puck as viewed in the lab frame. Discuss your reasoning with your partners.

✓ **STOP HERE** and check your results with an instructor before proceeding to the next section.

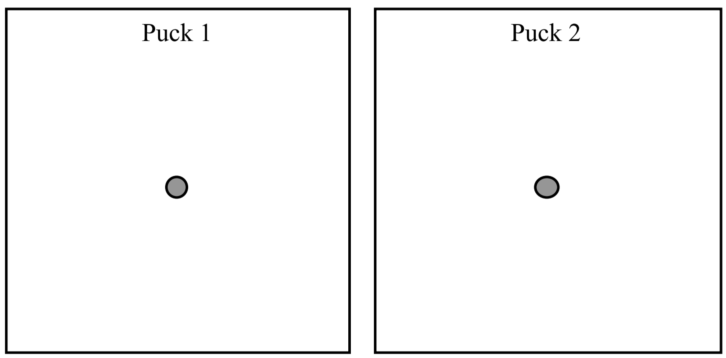
D. In parts A – C above we examined the motion of the pucks as the platform rotated counter-clockwise at a constant rate with respect to the lab frame. Below we consider other situations in which the rotational motion of the platform is different from before.

Top view diagram in platform frame
(Platform rotates in lab frame)

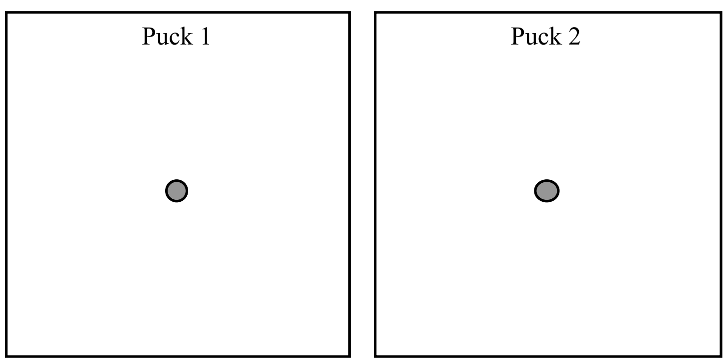


For each case below, draw free-body diagrams for pucks 1 and 2 *as measured in the platform frame.* (Assume that the initial velocities of the pucks as measured in the platform frame are the same as before.) Discuss your reasoning with your partners.

- The platform spins counter-clockwise *more quickly* than before (but still at a constant rate).



- The platform spins *clockwise* (rather than counter-clockwise) at a constant rate.



- The platform spins counter-clockwise with *decreasing* (not constant) angular speed.