Homework: Accelerating reference frames: The Foucault pendulum Name ________________________

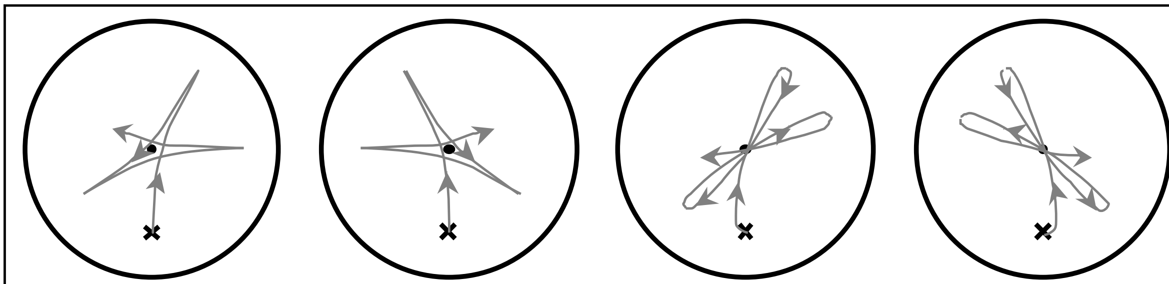
1. In the tutorial you identified which fictitious "force" is responsible for the precession of a Foucault pendulum when viewed in an Earthbound reference frame.

   For each of the other fictitious "forces," carefully explain why each one cannot be responsible for the precession. Draw diagrams to help make your explanations clear.

   (*Hints:* Which "force(s)" are equal to *zero,* or are *negligibly small* in magnitude? Which other "force(s)," though sizeable, have directions that do not adequately account for the precession?)

2. Consider in greater detail the motion of the pendulum observed from the frame of a rotating platform (from section I of the tutorial). In particular, examine the top view diagrams below. Each diagram illustrates a possible trajectory for the pendulum bob as observed in the platform frame.

   (Recall that the pendulum is suspended from a point directly above the center of the platform, and that the pendulum bob is released from rest as observed in the lab frame.)



(A) (B) (C) (D)

   a. Which trajectory best illustrates the motion of the pendulum for the case in which the platform rotates counter-clockwise at a constant rate (relative to the lab frame)? Explain your reasoning, and in your explanation be sure to address the following issues:

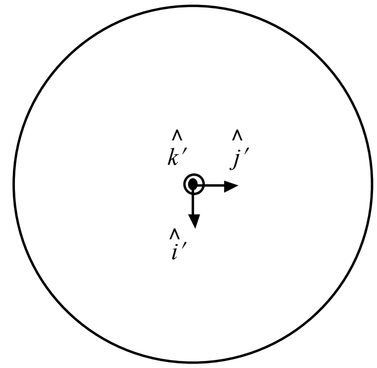
      - In the platform frame, is the initial velocity of the pendulum bob equal to *zero?*

      - In the platform frame, does the pendulum bob pass over the *exact center* of the platform?

   b. How would your answer to part a be different if the platform were rotating clockwise (rather than counter-clockwise) relative to the lab frame? Explain your reasoning.

3. Consider again the situation from section I of the tutorial, in which a (tall) simple pendulum undergoes small-amplitude oscillations above a platform that rotates counter-clockwise at a constant angular speed ω_o. The motion of the pendulum is observed in a reference frame that rotates with the platform and whose origin is fixed at the center. (See diagram at right.)

Top view diagram ***in platform frame***
(Platform spins CCW in lab frame)



Note: As before, assume that the frequency of rotation of the platform is <u>much smaller than</u> the frequency of the pendulum.

a. Briefly explain why taking small-amplitude oscillations leads to the approximation: $\dot{z}' \cong 0$.

b. In terms of $|\vec{\omega}_E|$, λ, $\dot{x}'$, and $\dot{y}'$, determine the x'- and y'-components of the Coriolis "force" as viewed in the platform frame. Show all work.

Hint: Recall that the cross product of vectors expressed in Cartesian coordinates can be calculated as a 3 × 3 determinant:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

c. Use your results from part b to complete the x'- and y'-component equations of motion for the pendulum bob as viewed in the platform frame.

(*Note:* The first term on the right-hand side of each equation represents the x'- or y'-component of the tension force exerted on the pendulum bob, which plays the role of the restoring force.)

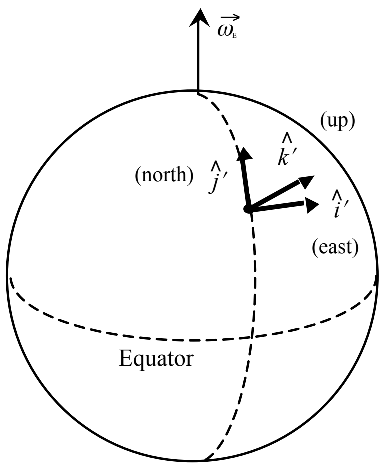
$$m\ddot{x}' = -\frac{mg}{L}x' + F_{fictitious,x'} = \ldots$$

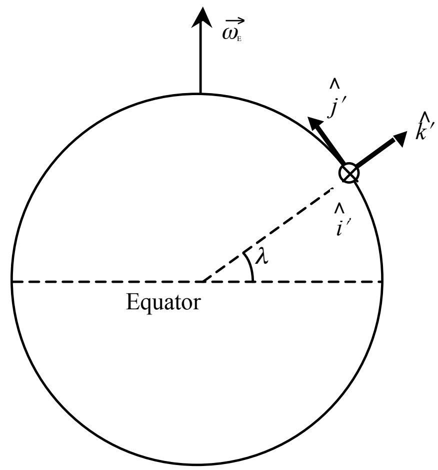
$$m\ddot{y}' = -\frac{mg}{L}y' + F_{fictitious,y'} = \ldots$$

d. <u>In your own words</u>, use your x'- and y'-component equations of motion from part c to explain which direction (whether *clockwise* or *counter-clockwise*) the pendulum will precess. Be sure to use <u>each</u> component equation of motion in your explanation.

(*Hint:* For example, for the x'-component equation of motion, choose an appropriate direction for the initial velocity of the pendulum bob, and explain in words how to use that equation to determine the direction of the subsequent "deflection" of the pendulum.)

4. Consider the small-amplitude motion of a Foucault pendulum at (northern) latitude λ. Let $\dot{x}'$ and $\dot{y}'$ represent the components of the velocity of the pendulum bob as measured in the Earthbound coordinate system illustrated below. (For small amplitudes we approximate: $\dot{z}' \cong 0$.)





a. Resolve the components of $\vec{\omega}_E$ in terms of $|\vec{\omega}_E|$, λ, and the primed unit vectors defined above. Clearly show all work.

b. In terms of $|\vec{\omega}_E|$, λ, $\dot{x}'$, and $\dot{y}'$, determine the x'- and y'-components of the Coriolis "force." Show all work. (*Note:* See hint from part b of Problem 3.)

c. Use your results from part b to complete the x'- and y'-component equations of motion for the pendulum bob as viewed in the Earthbound frame.

$$m\ddot{x}' = -\frac{mg}{L}x' + F_{fictitious,x'} = \ldots$$

$$m\ddot{y}' = -\frac{mg}{L}y' + F_{fictitious,y'} = \ldots$$

d. <u>In words</u>, use the x'- and y'-component equations of motion to explain which direction (whether *clockwise* or *counter-clockwise*) a pendulum will precess if it were located in the northern hemisphere ($\lambda > 0$). Be sure to use <u>each</u> component equation of motion in your explanation.

e. Repeat part d, except consider now a pendulum located in the southern hemisphere ($\lambda < 0$).

f. Compare and contrast your component equations from part c of this problem to those from part c of Problem 3. In particular:

   i. How does the value of λ affect the x'- and y'-components of the net force on the pendulum?

   ii. A Foucault pendulum at the North Pole would require 24 hours ($2\pi/\omega_E$) to precess. How many hours would be required for a Foucault pendulum <u>at your latitude</u>? Explain how you can tell from inspecting and comparing both sets of equations of motion.

5. At which location(s) on Earth, if any, would a Foucault pendulum of length L precess:

   a. clockwise at a maximum possible rate?

   b. counter-clockwise at a maximum possible rate?

   c. not at all?

   d. with a precession rate half as large as what it would have at your geographic location?

   e. with a precession rate twice as large as what it would have at your geographic location?

   Explain your reasoning in each case.

