Because physics majors have conceptual difficulties too:

Development of a tutorial approach to teaching intermediate mechanics

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Outline of joint talk

- Introduction
- Probing how students infuse physics meaning into mathematics
  - *Example #1: Velocity-dependent forces*
- Probing how students extract physics meaning from mathematics
  - *Example #2: Conservative force fields*
- Conclusions

From previous research at the introductory level

After standard lecture instruction in introductory physics, most students:*

– lack a *functional understanding* of many basic physical concepts

(*i.e.,* they lack the ability to apply a concept in a context different from that in which the concept was introduced)

– lack a coherent framework relating those concepts

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* McDermott and Redish, "Resource letter PER-1: Physics Education Research," Am. J. Phys. **67** (1999).

What is “intermediate mechanics” about?

Review of fundamental topics

- Vectors
- Kinematics
- Newton’s laws
- Work, energy, energy conservation
- Linear and angular momentum

New applications and extensions

- Velocity-dependent forces
- Linear and non-linear oscillations
- Conservative force fields
- Non-inertial reference frames
- Central forces, Kepler’s laws

New formalism and representations

- Scalar and vector fields; del operator; gradient, curl
- Phase space diagrams

As an *instructor* of intermediate mechanics

One might expect students to have already developed:

- *functional understanding* of physical concepts covered at the introductory level

- mathematical and reasoning skills necessary to extend those concepts in solving more sophisticated problems, *both qualitative and quantitative*

As a *physics education researcher* teaching intermediate mechanics

We might think about the following research questions:

- To what extent have students developed a functional understanding of fundamental concepts in mechanics?
- What unexpected things are students doing as they encounter new topics in intermediate mechanics?
- How is the use of mathematics different in this course than in the introductory courses?

Context of investigation and curriculum development

Primary student populations: Intermediate mechanics

- Grand Valley State University (GVSU)
- University of Maine (U. Maine)
- Seattle Pacific University (SPU)
- Pilot sites for *Intermediate Mechanics Tutorials*

Primary research methods

- Ungraded quizzes (pretests)
- Written examinations

} ***"Explain your reasoning."***

- Formal and informal observations in classroom
- Individual and group student clinical interviews

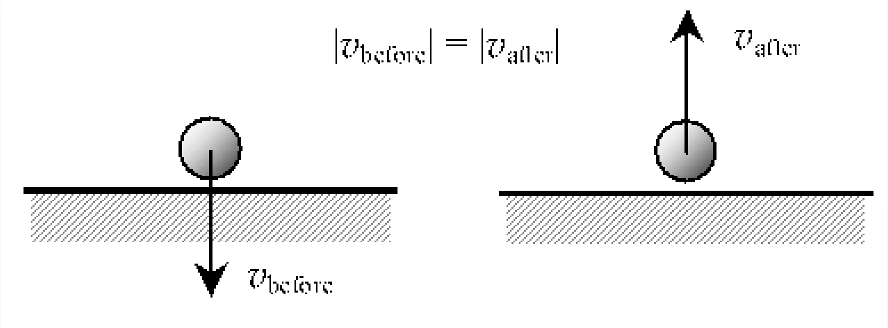
Example #1

Probing how students infuse physics meaning into mathematics

Velocity-dependent forces

What we might expect of our students in intermediate mechanics

- Recognize when and how to utilize skills (*e.g.*, draw free-body diagrams) and principles (*e.g.*, apply Newton's Second law) from introductory mechanics
  - *Task:* Is the acceleration of the ball larger *before* or *after* it bounces off the floor?



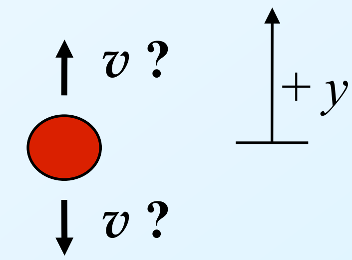
- Translate information about force and motion into correct (differential) equations of motion

Equations of motion involving air resistance

Midterm exam, GVSU, 2001 & 2003 ($N = 13$)

Take vertically upward to be the positive direction . For each equation below, determine whether that equation could apply to:

(a) a situation in which an object moves *upward,*
(b) a situation in which an object moves *downward,*
(c) *either* of these, or (d) *neither* of these.



Explain your reasoning for each case.

i) $ma = -mg + c_1 v$ **neither** "moving downward"

ii) $ma = -mg - c_1 v$ **either** "moving upward"

iii) $ma = -mg + c_2 v^2$ **moving downward**

iv) $ma = -mg - c_2 v^2$ **moving upward**

Sign of $c_1 v$ force "hardwired" into equation

(5/13)

Listening to Student Reasoning Using Interviews

A single student provides insight into a mistake made by a majority.

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* Hayes, Wittmann, "The role of sign in students' modeling signs of scalar equations," accepted for publication in The Physics Teacher. Expected publish date, Fall, 2009..

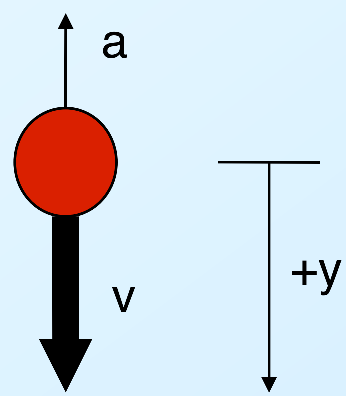
Velocity Dependent Forces in Vertical Situations

A ball is thrown vertically downward at greater than terminal velocity. It experiences an air resistance force proportional to *v.* Find an equation that describes the velocity of the ball with respect to time. Let +y be in the downward direction.

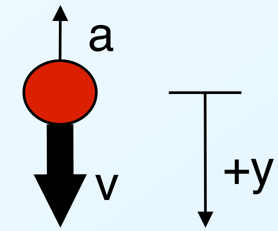
$$ma = mg - c_1 v$$

$$m\, dv/dt = mg - c_1 v$$

$$-ma = mg - c_1 v$$



Matching the physical system to the coordinate system



Correct Student Reasoning:

Starting with: $ma = mg - c_1 v$

Because $mg < c_1 v$, $(mg - c_1 v)$ is a negative number

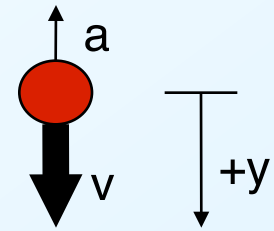
That gives: ma = negative number

Operates: a = negative number/m

Leaving: a = negative number

Consistent! We know *a* points upward (negative)

Cascading errors: Choice of sign causes revision of Newton's 2nd Law



ASSUMES *a* > 0:

Return to original equation:

$$\mathbf{ma = mg - c_1 v}$$

"So, using this equation it implies that ***a*** is positive. There's no negative in there. It assumed that ***a*** is positive."

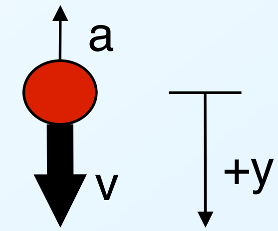
DERIVES *v* < 0:

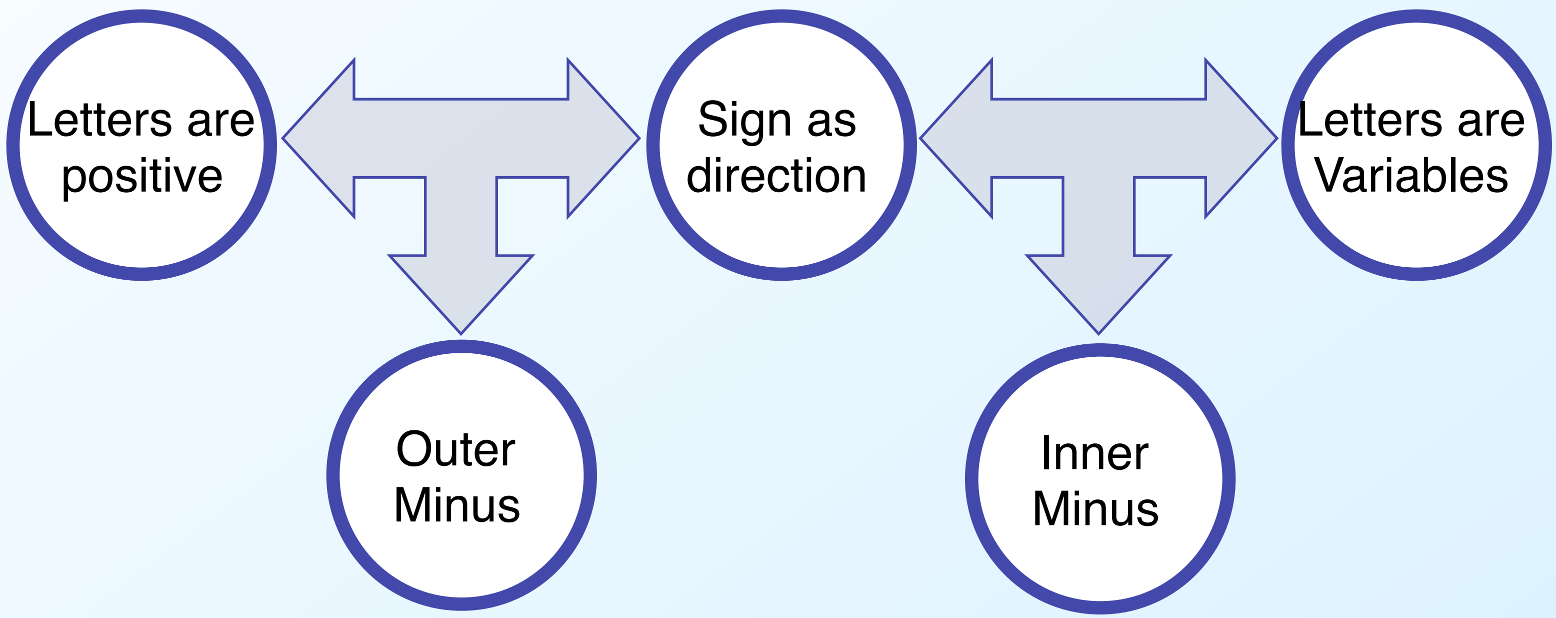
With *v* pointed upward, we have a contradiction!

Student places a minus sign in front of *ma*–term to take care of the direction error:

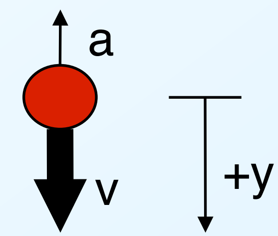
$$\mathbf{-\,ma = mg - c_1 v}$$

A collection of good ideas, combined in problematic ways





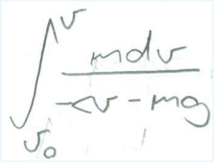
Consistencies with intro physics?



Yes.

g

Sign errors on a similar problem (Exam question, high stakes)



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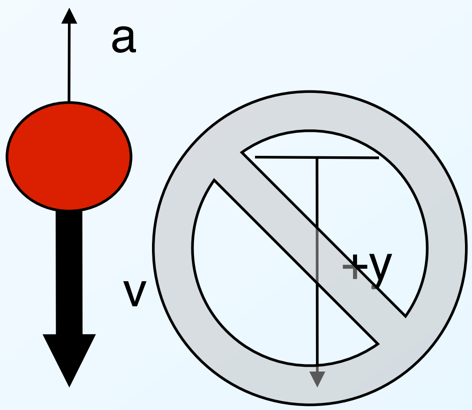

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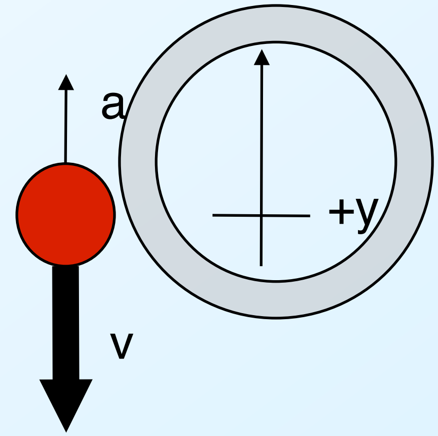


LOOKING FOR: $ma = -mg - c_1 v$
and integrate…

* Black, Wittmann, "Understanding the use of two integration methods on separable first order differential equations," under review at Physical Review Special TopicsPhysics Education Research. Pre-print available online at http://arxiv.org/abs/0902.0748.

$$\int_{v_o}^{v} \frac{m\,dv}{-cv - mg}$$

Sign errors in student solutions

$$m\frac{dv}{dt} = mg - cV \longrightarrow \int_{V_o}^{V} \frac{m\,dv}{mg - cV} = \int_0^t dt$$

Force due to Earth is down – negative

The initial speed is v_o but the direction is down so it should be $-v_o$.

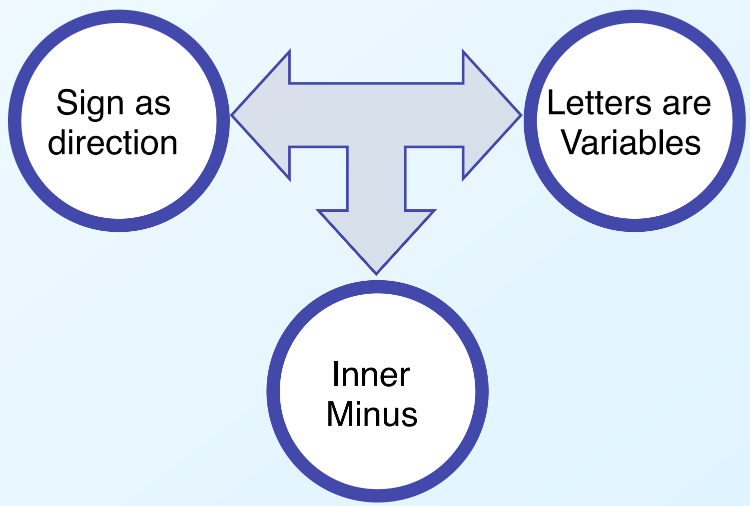
This term is negative and cannot lead to natural log solution in one easy step.

Assuming the the lower limit of integration is positive

$$\int_{v_0}^{v} \frac{m\,dv}{-cv - mg}$$

A constant gets treated like a variable

(allowing for an "inner minus")



$$\int_{v_0}^{v} \frac{m\,dv}{-cv - mg}$$

Assuming the the lower limit of integration is positive

$$\int_{v_0}^{v} \frac{m\,dv}{mg - cv}$$

$$\int_{v_0}^{v} \frac{m}{-mg + cv}\, dv$$

$$\int_{v_0}^{v} \frac{m\,dv}{-cv - mg}$$

$$\int_{v_0}^{v} \frac{m\,dv}{-cv - mg}$$

$$\int_{v_0}^{v} dv$$

$$\int_{v_0}^{v} \frac{du}{u}$$

Assuming the initial condition is positive

$$\int_{v_0}^{v} \frac{m\,dv}{-cv - mg}$$

$$V(t=0) = V_0$$

$$\dot{x}(0) = v_0$$

Three other students used either no initial value at all or *very odd* solutions with implied negative v_0

Student problem solving about velocity-dependent forces

Valuable ideas:

- *Sign defines direction* when mapping to a coordinate system
- *Letters represent variables* and describe functions

Contradictory ideas:

- *Letters represent variables* even when referring to constants
- *Letters represent constants*, so the problem-solver must fix mathematical statements to match the choice of coordinate system.

Example #2

Probing how students extract physics meaning from mathematics

Conservative force fields

What we teach about conservative forces

in intermediate mechanics

A force $\vec{F}(\vec{r})$ is conservative if and only if:

- the work by that force around any closed path is zero
- $\vec{\nabla} \times \vec{F} = 0$ at all locations
- a potential energy function $U(\vec{r})$ exists so that $\vec{F} = -\vec{\nabla} U$

(generalization of $\vec{E} = -\vec{\nabla} V$ from electrostatics)

A common theme from physics education research in introductory physics

Many students have difficulty discriminating between a **quantity** and its **rate of change**, for example:

- position *vs.* velocity *
- velocity *vs.* acceleration (or change in velocity) *
- height *vs.* slope of a graph **
- electric field *vs.* electric potential †
- electric charge *vs.* electric current
- *…and many other examples*

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* Trowbridge and McDermott, Am. J. Phys. **48** (1980) and **49** (1981); Shaffer and McDermott, Am. J. Phys. **73** (2005).

** McDermott, Rosenquist, and van Zee, Am. J. Phys. **55** (1987).

† Allain, Ph.D. dissertation, NCSU, 2001; Maloney *et al.,* Am. J. Phys. Suppl. **69** (2001).

"Equipotential map" pretest

Intermediate mechanics

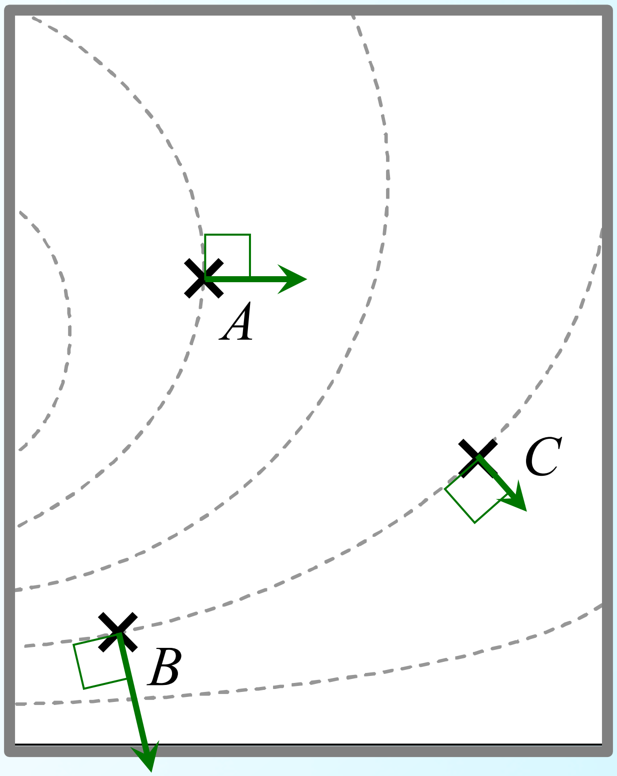
After all lecture instruction in introductory E&M

In the region of space depicted at right, the dashed curves indicate locations of *equal potential energy* for a test charge $+q_{\text{test}}$ placed within this region.

It is known that the potential energy at location *A* is *greater than* that at *B* and *C.*

A. At each location, draw an arrow to indicate the <u>direction</u> in which the test charge $+q_{\text{test}}$ would move when released from that location. Explain.

B. Rank the locations *A, B,* and *C* according to the <u>magnitude</u> of the force exerted on the test charge $+q_{\text{test}}$. Explain your reasoning.



(Qualitatively correct force vectors are shown.)

Equipotential map pretest: Results

Intermediate mechanics, GVSU (N = 73, 8 classes)

After all lecture instruction in introductory E&M

Percent correct *with correct reasoning:*
(rounded to nearest 5%)

Part A (Directions of force vectors)	50%	(35/73)
Part B (Ranking force magnitudes)	20%	(14/73)
Both parts correct	**15%**	**(9/73)**

Similar results have been found among students at U. Maine, SPU, and pilot test sites.

Equipotential map pretest: Results

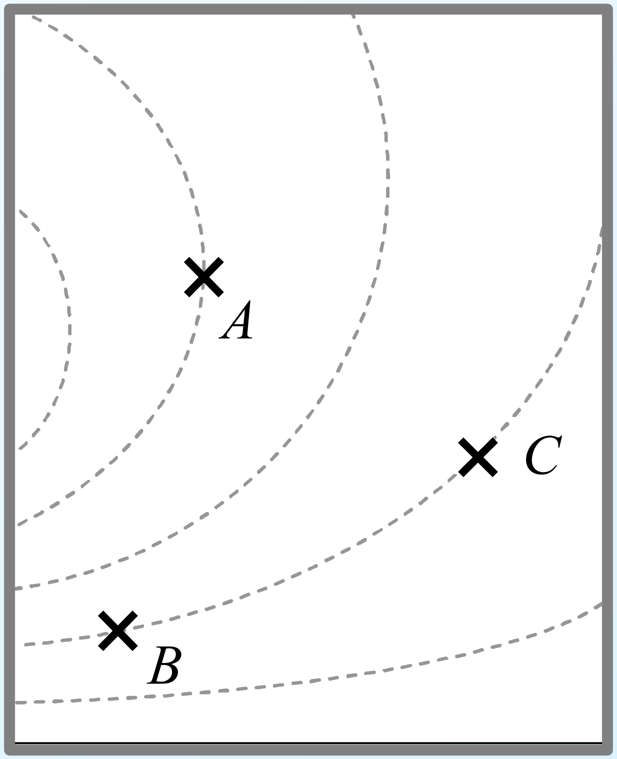
Intermediate mechanics

After all lecture instruction in introductory E&M

Most common *incorrect* ranking: $F_A > F_B = F_C$

Example: "A has the highest potential so it can exert a larger force on a test charge. B and C are on the same potential curve and thus have equal abilities to exert force."

Example: "A has the most potential pushing the charge fastest. B & C are on the same level."



Failure to discriminate between a quantity *(potential energy U)* ***and its rate of change*** *(force* $\vec{F} = -\vec{\nabla} U$*)*

Equipotential map pretest: Results

Intermediate mechanics

After all lecture instruction in introductory E&M

Most common *incorrect* ranking: $F_A > F_B = F_C$

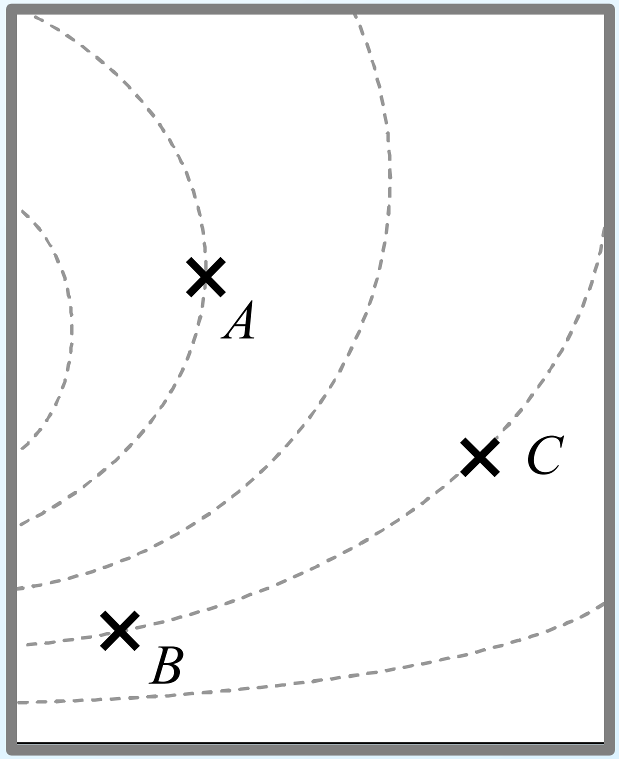
Example: "Since F is proportional to V, higher V means higher F."

Example:

"[$V_A > V_B = V_C$] … $F(x) = - dV/dx$

$\therefore$ $F_C = F_B$ in magnitude and

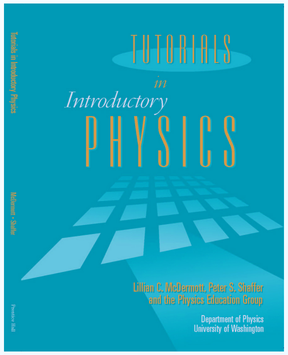
$F_A > F_C$ in magnitude."

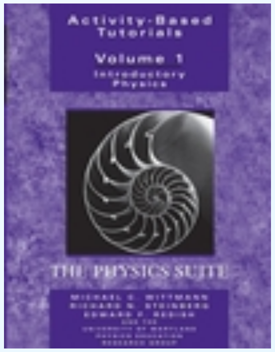


***Failure to discriminate between a quantity** (potential energy U)*

***and its rate of change** (force $\vec{F} = -\vec{\nabla} U$)*

A tutorial approach for teaching *introductory* mechanics





- Emphasis:
  - conceptual understanding and reasoning skills
  - integrating the mathematical formalism
- Tutorial components:
  - pretests (ungraded quizzes, in-class or take-home; 10 min)
  - tutorial worksheets (small-group work; 40 – 50 min)
  - tutorial homework
  - examination questions (post-tests)

Intermediate Mechanics Tutorials

Collaboration between GVSU (Ambrose)* and U. Maine (M. Wittmann)

- Simple harmonic motion
- Newton's laws and velocity-dependent forces
- Damped harmonic motion
- Driven harmonic motion
- Phase space diagrams
- Conservative force fields
- Harmonic motion in two dimensions
- Accelerating reference frames
- Orbital mechanics
- Generalized coordinates and Lagrangian mechanics

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* Ambrose, "Investigating student understanding in intermediate mechanics: Identifying the need for a tutorial approach to instruction," *Am. J. Phys.* **72** (2004).

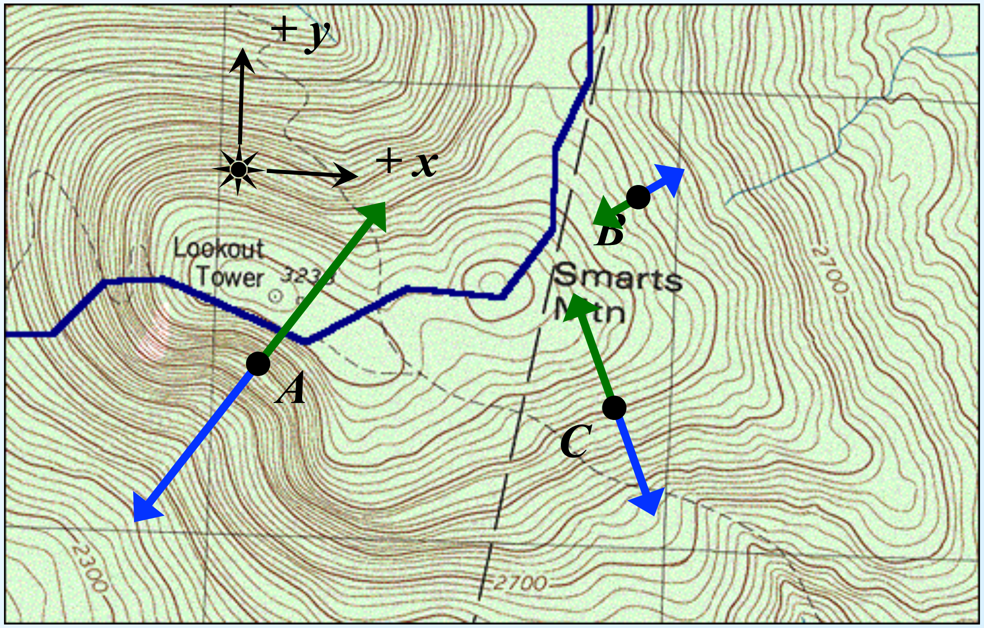
Building students' physical ***and*** mathematical intuitions about conservative forces

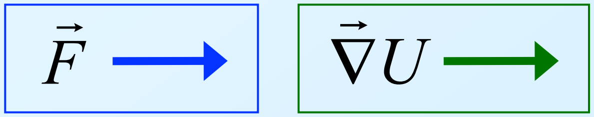
In the tutorial *Conservative forces and equipotential diagrams:*

Students develop a qualitative relationship between **force vectors** and local **equipotential contours**…

…and construct an **operational definition of the gradient** of potential energy:

$$\vec{\nabla} U = \left(\frac{\partial U}{\partial x} \hat{i} + \frac{\partial U}{\partial y} \hat{j} \right)$$



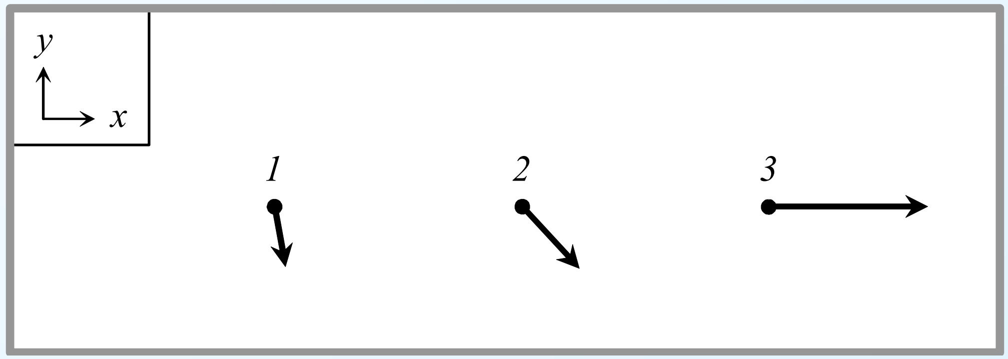


“Unknown equipotentials” post-test

Exam after tutorial, GVSU, 2003 ($N = 7$)

Three identical particles are located at the labeled locations *(1, 2, and 3)*.

Each vector represents the force $\boldsymbol{F}(x, y)$ exerted at that location, with:



$$F_3 > F_2 > F_1$$

A. In the space above, *carefully sketch an equipotential diagram* for the region shown. Make sure your equipotential lines are consistent with the force vectors shown. Explain the reasoning you used to make your sketch.

B. On the basis of your results in part A, rank the labeled locations according to the *potential energy* of the particle at that location. Explain how you can tell.

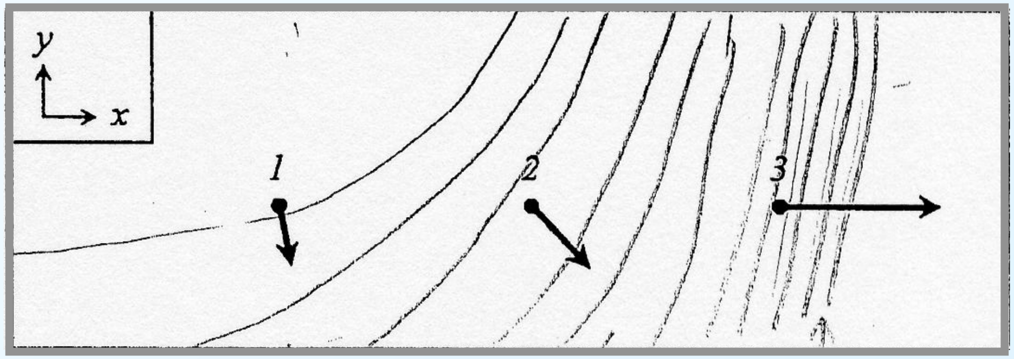
“Unknown equipotentials” post-test: Results

Exam after tutorial, GVSU, 2003 ($N = 7$)

Three identical particles are located at the labeled locations *(1, 2, and 3)*.

Each vector represents the force $\boldsymbol{F}(x, y)$ exerted at that location, with:

$$F_3 > F_2 > F_1$$



Acceptable student diagram *(part A)*

Part A: Relative spacing of equipotentials: **4/7 correct**

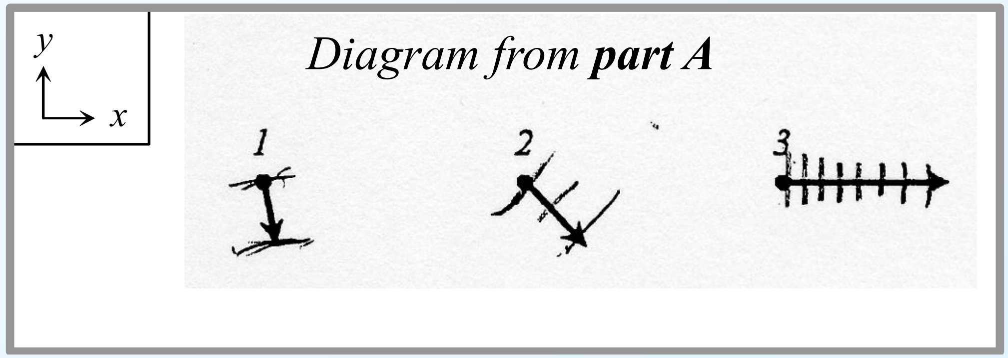
Orientation of equipotentials: **5/7 correct**

Part B: Rank points by potential energy: **1/7 correct**

“Unknown equipotentials” post-test: Results

Exam after tutorial, GVSU, 2003 ($N = 7$)

Example of a partially correct response:



Part B *(rank points by potential energy):* 3 > 2 > 1

The greater the force, the higher potential energy $\vec{F} = -\nabla V$

Persistent *confusion between a quantity (potential energy U) and its rate of change (force* $\vec{F} = -\vec{\nabla} U$*)*

Helping students understand what the gradient *means* <u>and</u> what it *does not mean*

Last page of tutorial includes these questions:

Summarize your results: Does $\vec{\nabla} U$ ...

- point in the direction of *increasing* or *decreasing* potential energy?
- point in the direction in which potential energy changes the *most* or the *least* with respect to position?
- ▶ have the *same magnitude* at all locations having the *same potential energy?* Explain why or why not.

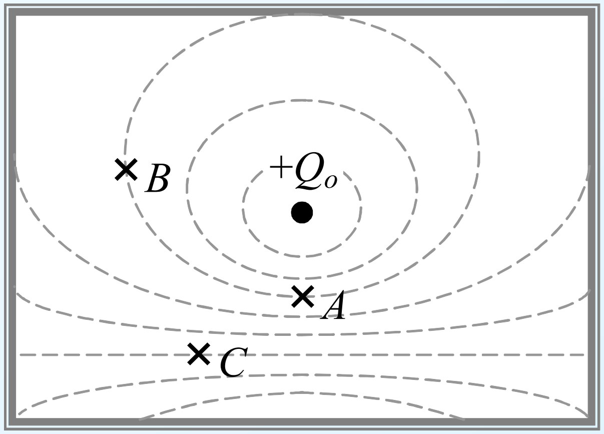
Examples of assessment questions

On written exams <u>after</u> tutorial instruction

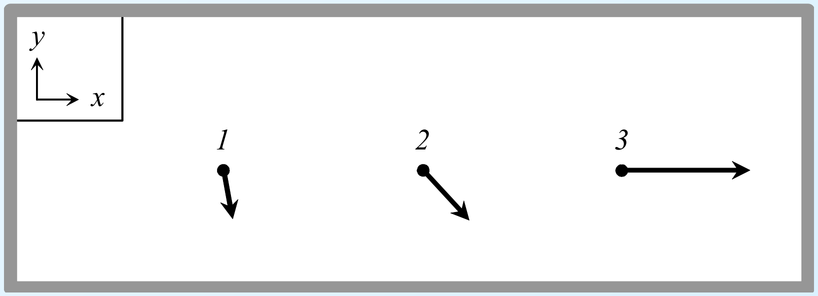
Task: Given equipotential map, predict directions and relative magnitudes of forces.

GVSU: **20/23 correct** (2 classes)

SPU: **8/11 correct** (1 class)



Task: Given several forces, sketch a possible equipotential map and rank points by potential energy.



GVSU: **14/30 correct** (3 classes)

Preliminary conclusions

- Intermediate mechanics students often experience conceptual and reasoning difficulties similar to those identified at the introductory level.
  - Confusion between a *quantity* and its (temporal or spatial) *rate of change*
  - Confusion between the meanings of *functions* and *variables*

Traditional instruction, even in advanced topics,
does not address basic difficulties.

Preliminary conclusions

- Specific conceptual and reasoning difficulties must be addressed *explicitly* and *repeatedly,* including those involving:
  - basic physics concepts
  - higher order physics concepts
  - the use of mathematics to describe physics
  - the use of mathematics to learn new physics

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Intermediate Mechanics Tutorials

Project website: **http://perlnet.umaine.edu/IMT**

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