Because physics majors have conceptual difficulties too:

Development of a tutorial approach to teaching intermediate mechanics

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Outline of presentation

- Introduction and motivation for project
- Investigating the nature of student thinking: Probing ability of students to extract physical meaning from mathematics
  - *Example #1:* Conservative force fields
  - *Example #2:* Harmonic oscillations in 1D and 2D
- Using research to design and assess a tutorial approach to teaching intermediate mechanics
- Reflections and conclusions

From previous research at the introductory level

Many students have difficulty discriminating between a **quantity** and its **rate of change:**

- position *vs.* velocity*
- velocity *vs.* acceleration *
- height *vs.* slope of a graph **
- electric field *vs.* electric potential †
- electric (or magnetic) flux *vs.* change in flux
- *…and many other examples*

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* Trowbridge and McDermott, Am. J. Phys. **48** (1980) and **49** (1981); Shaffer and McDermott, Am. J. Phys. **73** (2005).

** McDermott, Rosenquist, and van Zee, Am. J. Phys. **55** (1987).

† Allain, Ph.D. dissertation, NCSU, 2001; Maloney *et al.,* Am. J. Phys. Suppl. **69** (2001).

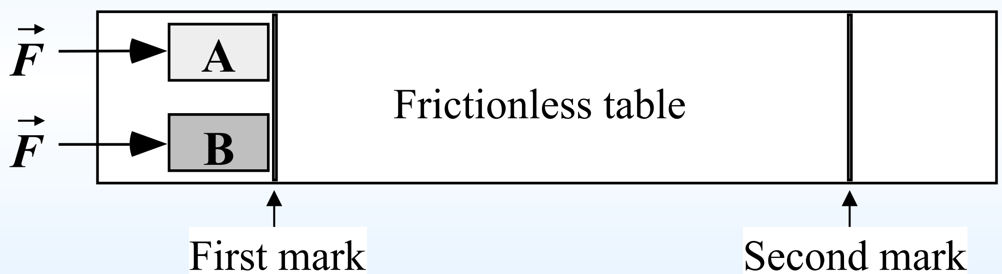
From previous research at the introductory level

Students use inappropriate "**compensation arguments**" when comparing quantities that involve two or more variables.

Example: Two carts, $m_A < m_B$, are at rest on a level, frictionless table.

Equal forces are exerted on the carts as they move between the two marks.*

Top view diagram



Students often (incorrectly) predict:

"$KE_A > KE_B$" because faster speed of A "matters more" than mass ($KE = ½mv^2$)

"$p_A = p_B$" because larger mass of B "compensates for" smaller speed ($p = mv$)

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* R.A. Lawson and L.C. McDermott, *Am. J. Phys.* **55**, 811-817 (1987).

From previous research at the introductory level

After standard lecture instruction in introductory physics, most students:*

- lack a *functional understanding* of many basic physical concepts

  (*i.e.,* they lack the ability to apply a concept in a context different from that in which the concept was introduced)

- lack a coherent framework relating those concepts

* McDermott and Redish, "Resource letter PER-1: Physics Education Research," Am. J. Phys. **67** (1999).

What is "intermediate mechanics" about?

Review of fundamental topics

- Vectors
- Kinematics
- Newton's laws
- Work, energy, energy conservation
- Linear and angular momentum

New applications and extensions

- Velocity-dependent forces
- Linear and non-linear oscillations
- Conservative force fields
- Non-inertial reference frames
- Central forces, Kepler's laws

New formalism and representations

- Scalar and vector fields;
  del operator; gradient, curl
- Phase space diagrams

As an *instructor* of intermediate mechanics

One might expect students to have already developed:

- *functional understanding* of physical concepts covered at the introductory level

- mathematical and reasoning skills necessary to extend those concepts in solving more sophisticated problems, *both qualitative and quantitative*

As a *physics education researcher* teaching intermediate mechanics

We might think about the following research questions:

- To what extent have students developed a functional understanding of fundamental concepts in mechanics?
- What unexpected things are students doing as they encounter new topics in intermediate mechanics?
- How is the use of mathematics different in this course than in the introductory courses?

Context of investigation and curriculum development

Primary student populations: Intermediate mechanics

- Grand Valley State University (GVSU)
- University of Maine (U. Maine)
- Seattle Pacific University (SPU)
- Pilot sites for *Intermediate Mechanics Tutorials*

Primary research methods

- Ungraded quizzes (pretests)
- Written examinations

} ***"Explain your reasoning."***

- Formal and informal observations in classroom
- Individual and group student clinical interviews

Example #1

Conservative forces

What we teach about conservative forces

in intermediate mechanics

A force **F(r)** is conservative if and only if:

- the work by that force around any closed path is zero
- $\nabla \times \mathbf{F} = 0$ at all locations
- a potential energy function $U(\mathbf{r})$ exists so that $\mathbf{F} = -\nabla U$

(generalization of $\mathbf{E} = -\nabla V$ from electrostatics)

"Equipotential map" pretest

Intermediate mechanics

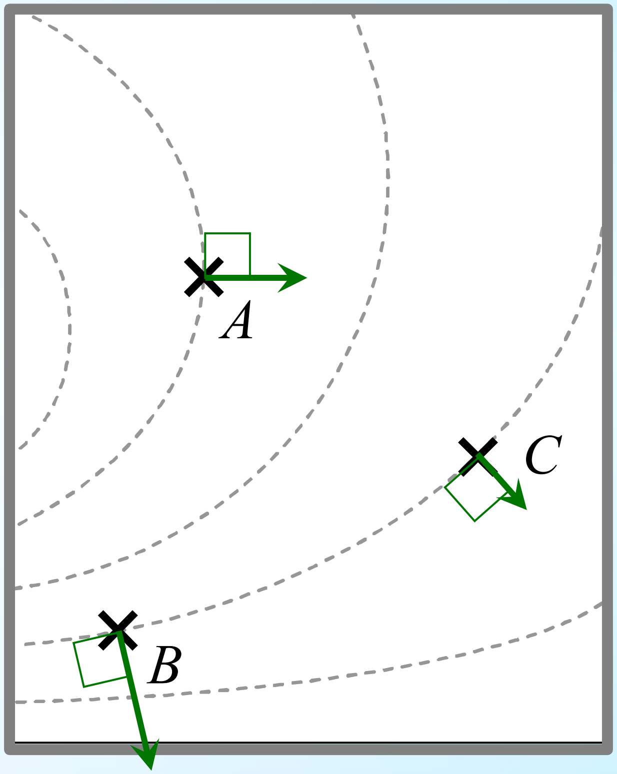
After all lecture instruction in introductory E&M

In the region of space depicted at right, the dashed curves indicate locations of *equal potential energy* for a test charge $+q_{test}$ placed within this region.

It is known that the potential energy at location *A* is *greater than* that at *B* and *C*.

A. At each location, draw an arrow to indicate the direction in which the test charge $+q_{test}$ would move when released from that location. Explain.

B. Rank the locations *A, B,* and *C* according to the magnitude of the force exerted on the test charge $+q_{test}$. Explain your reasoning.



(Qualitatively correct force vectors are shown.)

Equipotential map pretest: Results

Intermediate mechanics, GVSU (N = 73, 8 classes)

After all lecture instruction in introductory E&M

Percent correct *with correct reasoning:*
(rounded to nearest 5%)

Part A (Directions of force vectors)	50%	(35/73)
Part B (Ranking force magnitudes)	20%	(14/73)
Both parts correct	**15%**	**(9/73)**

Similar results have been found among students at U. Maine and pilot test sites.

Equipotential map pretest: Results

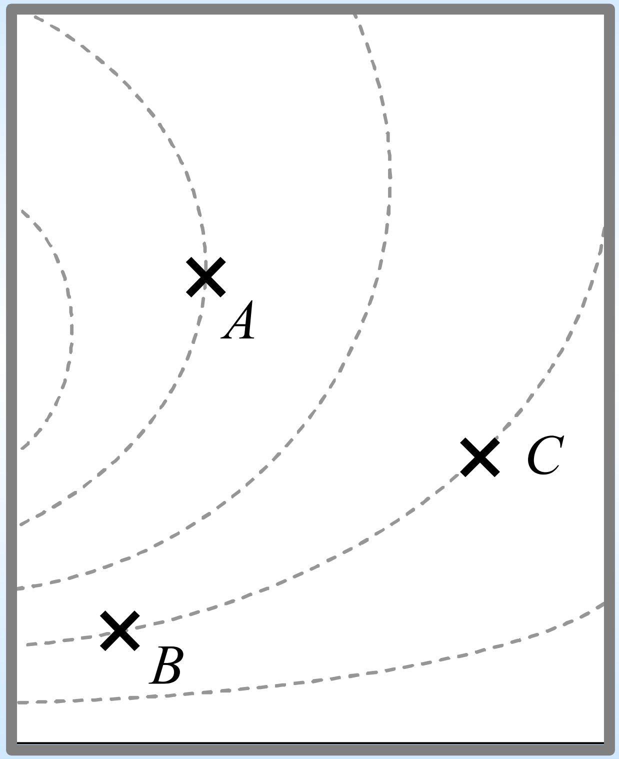
Intermediate mechanics

After all lecture instruction in introductory E&M

Most common *incorrect* ranking: $F_A > F_B = F_C$

Example: "A has the highest potential so it can exert a larger force on a test charge. B and C are on the same potential curve and thus have equal abilities to exert force."

Example: "A has the most potential pushing the charge fastest. B & C are on the same level."



***Failure to discriminate between a quantity** (potential energy U)*
***and its rate of change** (force $\mathbf{F} = -\nabla U$)*

Equipotential map pretest: Results

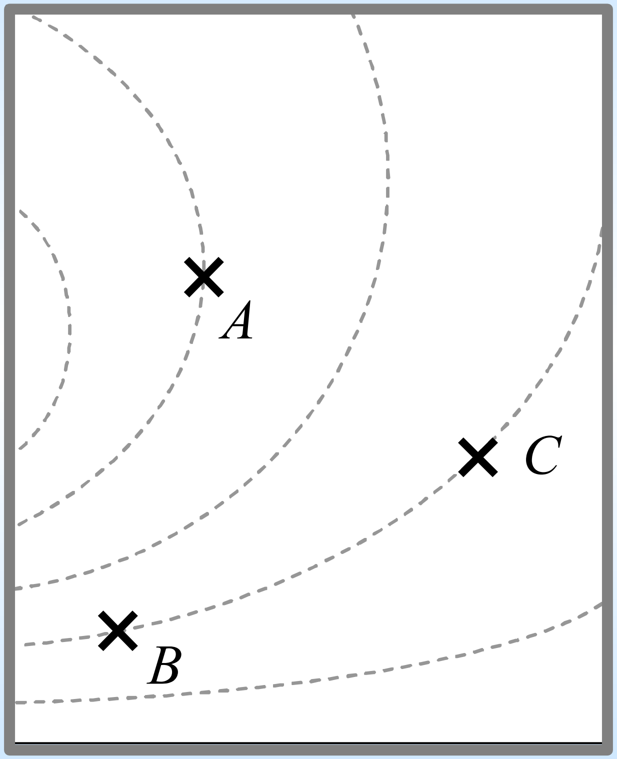
Intermediate mechanics

After all lecture instruction in introductory E&M

Most common *incorrect* ranking: $F_A > F_B = F_C$

Example: "Since F is proportional to V, higher V means higher F."

Example:
"[$V_A > V_B = V_C$] … $F(x) = - dV/dx$
$\therefore$ $F_C = F_B$ in magnitude and
$F_A > F_C$ in magnitude."



Failure to discriminate between a quantity *(potential energy U)* ***and its rate of change*** *(force* $\mathbf{F} = -\nabla U$*)*

Example #2

Harmonic motion in 1D and 2D

Research questions

After relevant lecture instruction:

- How well do students understand the factors that affect the frequency of harmonic oscillations?
  - Simple harmonic motion in 1-D and 2-D
  - Damped harmonic motion
- How well do students use and interpret formal representations of oscillatory motion?
  - Motion graphs of 1D oscillators
  - Phase space diagrams of 1D oscillators
  - Real space *(x-y)* trajectories of 2D oscillators

What we teach about harmonic oscillations

in intermediate mechanics

	Equation of motion	Solution for *x(t)*
Simple harmonic motion	$m\ddot{x} = -kx$	$x(t) = A_o \cos(\omega_o t + \varphi)$ where $\boxed{\omega_o = \sqrt{k/m}}$
Underdamped motion $(\gamma < \omega_o)$	$m\ddot{x} = -kx - c\dot{x}$ $\left(\ddot{x} = -\omega_o^2 x - 2\gamma\dot{x}\right)$	$x(t) = A_o\, e^{-\gamma t} \cos(\omega_d t + \varphi)$ where $\omega_d = \sqrt{\omega_o^2 - \gamma^2}$

⇒ Frequency depends on mass and spring constant

⇒ Amplitude has no effect on frequency or period

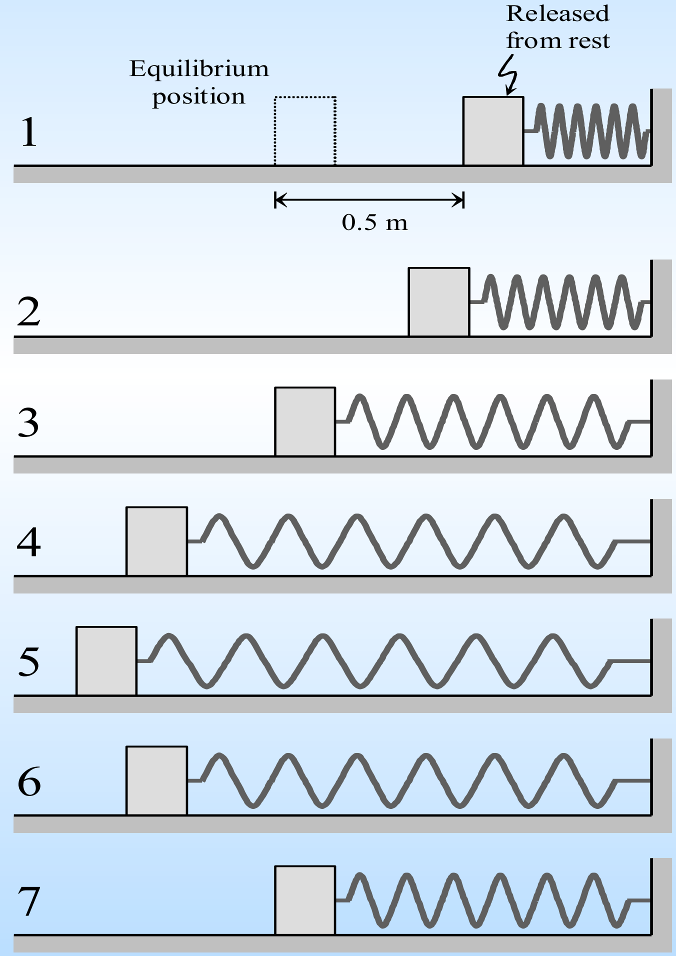
"Simple harmonic oscillator" pretest

(excerpt)

A block is connected to a spring and placed on a frictionless surface. A student releases the block <u>0.5 m</u> to the <u>right</u> of equilibrium.

For each change listed below, how (if at all) would that change affect the **period** of motion? Explain your reasoning.

- The block is released <u>0.7 m</u> to the <u>left</u> of equilibrium.
- The spring is replaced with a stiffer spring.
- The block is replaced with another block four times the mass as the original one.



Predicting effect on oscillation frequency

After lecture instruction (6 classes, $N \sim 50$)

The good news…

Parts ii & iii (changing ***spring*** or ***mass***):

Most students **(~ 65%)** gave qualitatively correct answers with acceptable explanations.

The bad news…

Part i (increasing ***amplitude***):

Most students answered correctly (~ 65%) but very few gave acceptable explanations.

Most common incorrect response **(~25%):**
"Larger [period if amplitude is larger], because the block travels farther during each period."

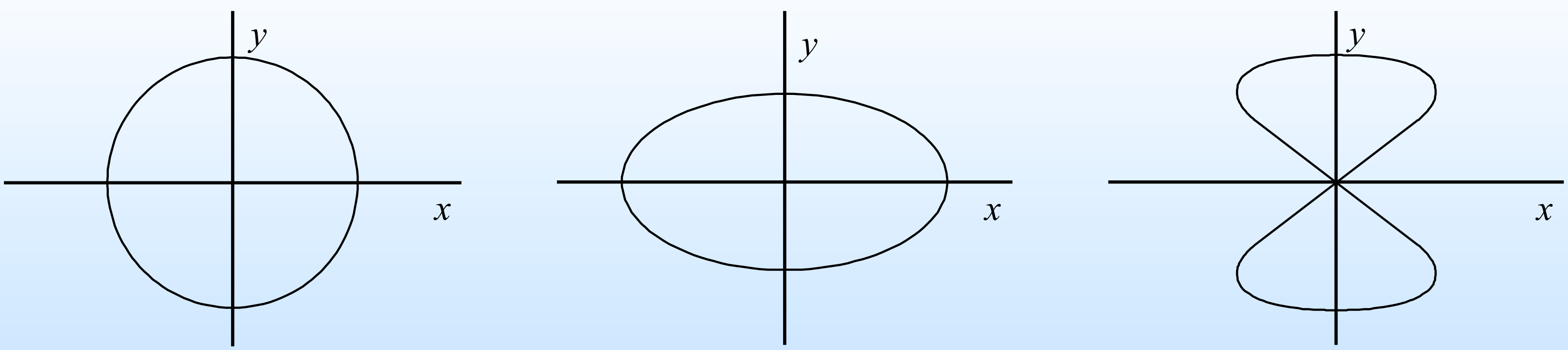
"2D oscillator" pretest

Consider the motion of a 2D oscillator, with $U(x, y) = \frac{1}{2}\, k_1 x^2 + \frac{1}{2}\, k_2 y^2$, or equivalently, $U(x, y) = \frac{1}{2}\, m\omega_1^2 x^2 + \frac{1}{2}\, m\omega_2^2 y^2$.

For each x-y trajectory shown, could the oscillator follow that trajectory?

If so: Is $\boldsymbol{\omega_1}$ *greater than, less than,* or *equal to* $\boldsymbol{\omega_2}$*?* Explain.*

If not: Explain why not.



Case #1 **Case #2** **Case #3**

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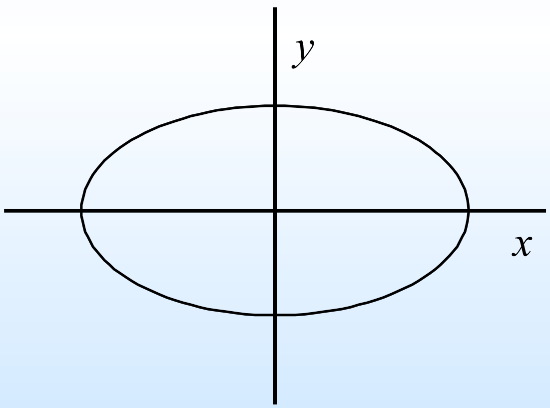
* Original version asked instead for a comparison between $\boldsymbol{k_1}$ and $\boldsymbol{k_2}$.

2D oscillator pretest: Results

Intermediate mechanics, GVSU (4 classes) and U. Maine (1 class)

***After** relevant lecture instruction*

- Few students (0% - 15%) answered all cases correctly.
- Most incorrect responses based on "compensation arguments" involving relative amplitudes along *x*- and *y*-axes:



Case #2

<u>*Examples:*</u>

"$k_1 < k_2$, the spring goes farther in the *x*-direction, so spring must be less stiff in that direction."

"$\omega_2 > \omega_1$. Since we now have an oval curve with the *x*-axis longer, ω_2 must be greater to compensate."

What we teach **about harmonic oscillations**

in intermediate mechanics

	Equation of motion	Solution for *x(t)*
Simple harmonic motion	$m\ddot{x} = -kx$	$x(t) = A_o \cos(\omega_o t + \varphi)$ where $\omega_o = \sqrt{k/m}$
Underdamped motion $(\gamma < \omega_o)$	$m\ddot{x} = -kx - c\dot{x}$ $\left(\ddot{x} = -\omega_o^2 x - 2\gamma\dot{x}\right)$	$x(t) = A_o e^{-\gamma t} \cos(\omega_d t + \varphi)$ where $\omega_d = \sqrt{\omega_o^2 - \gamma^2}$

⇒ Damping force causes amplitude to decrease over time

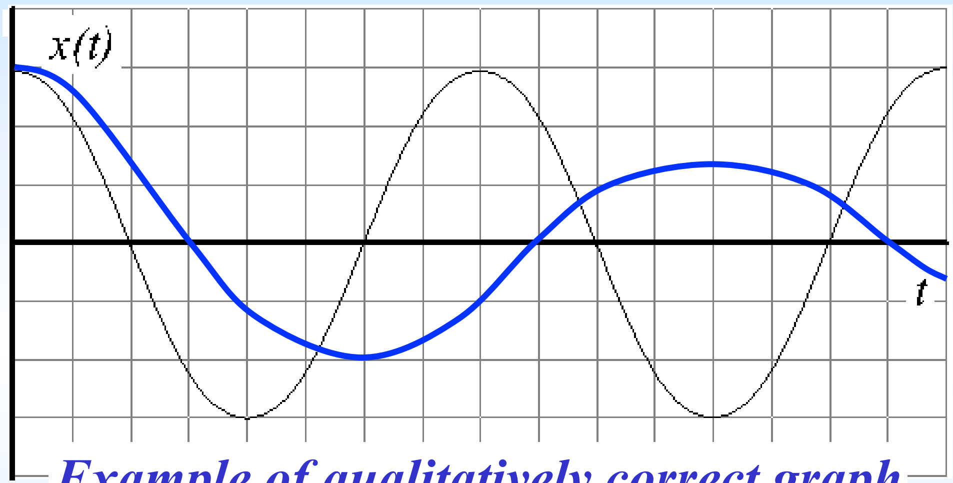
⇒ Damping force lowers oscillation frequency $(\boldsymbol{\omega_d < \omega_o})$

"Underdamped oscillator" pretest

(excerpt)

The *x* vs. *t* graph represents the motion of a simple harmonic oscillator that is released from rest at $t = 0$.

A. Clearly indicate and label (i) amplitude, (ii) period. Explain your reasoning.



Example of qualitatively correct graph

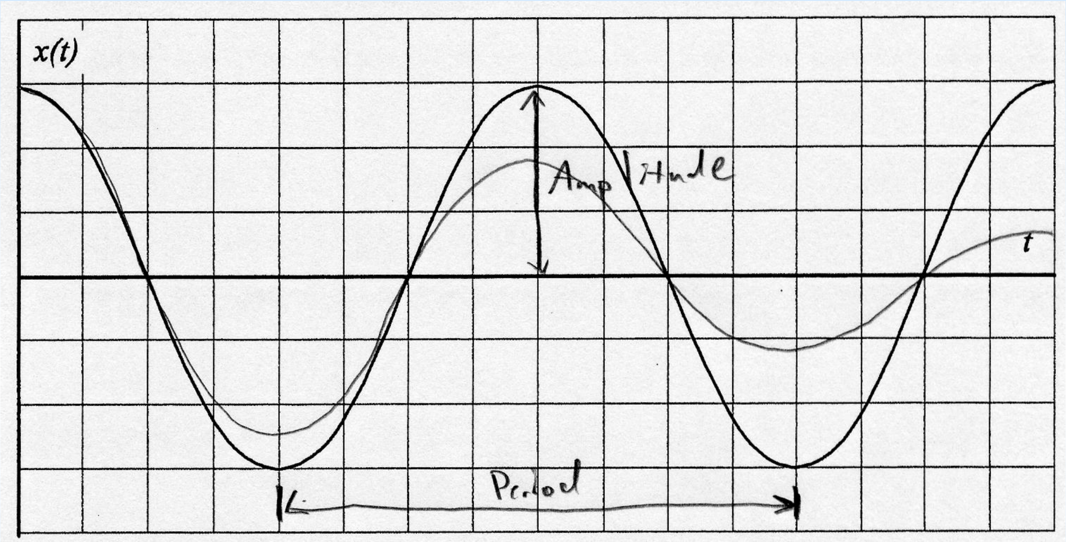
B. Suppose that a retarding force were applied to cause the oscillator to become <u>underdamped</u>.

On the axes above, sketch a qualitatively correct *x* vs. *t* graph for the oscillator when it is released *from rest* at the *same initial position as before.* Explain how you decided to draw the graph the way you did.

Underdamped oscillator pretest: Results

After lecture instruction, GVSU (5 classes) and SPU (1 class)

Most common *incorrect* response (60% - 70%):



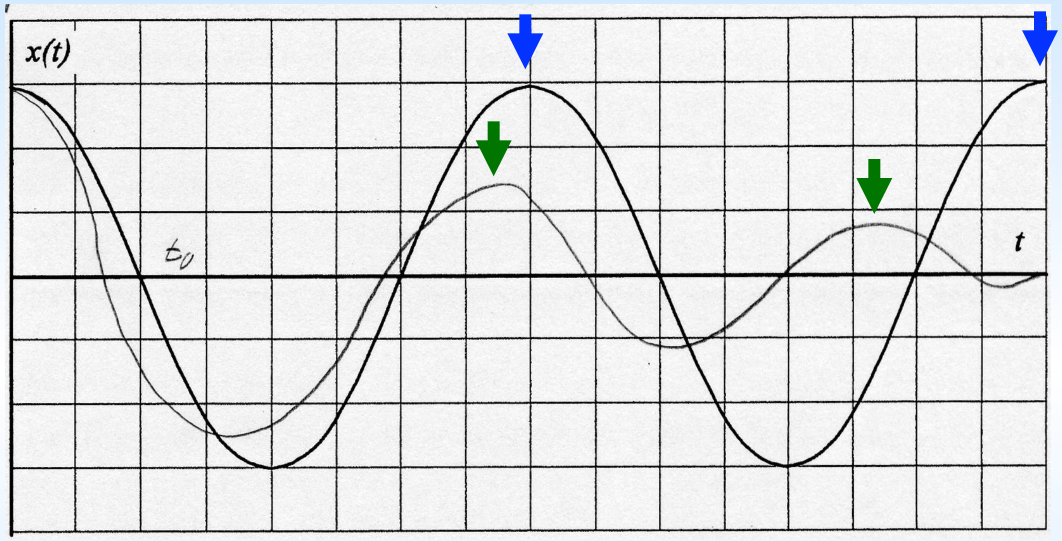
"The amplitude will shrink with time but the period shouldn't change since they are independent of each other."

Failure to recognize that damping force affects frequency

Underdamped oscillator pretest: Results

After lecture instruction, GVSU (5 classes) and SPU (1 class)

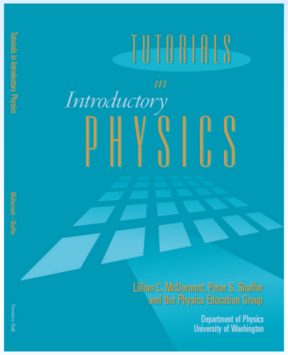
Other category of *incorrect* response:

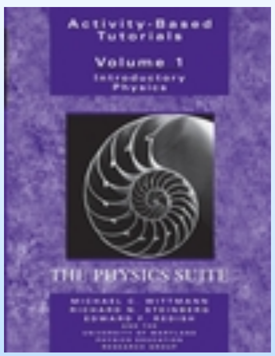


"The damping agent will prevent the oscillator to reach a certain point in the same amount of time, so the undamped motion 'leads' the damped motion."

Difficulty connecting verbal and graphical (x vs. t) representations of oscillatory motion

A research-tested tutorial approach for teaching *introductory* mechanics





- Emphasis:
  - conceptual understanding and reasoning skills
  - connecting the mathematics to the physics
- Tutorial components:
  - pretests (ungraded quizzes,
    in-class or take-home; 10 min)
  - tutorial worksheets
    (small-group work; 40 – 50 min)
  - tutorial homework
  - examination questions (post-tests)

Intermediate Mechanics Tutorials

Collaboration between GVSU (Ambrose)* and U. Maine (M. Wittmann)

- Simple harmonic motion
- Newton's laws and velocity-dependent forces
- Damped harmonic motion
- Driven harmonic motion
- Phase space diagrams
- Conservative force fields
- Harmonic motion in two dimensions
- Accelerating reference frames
- Orbital mechanics
- Generalized coordinates and Lagrangian mechanics

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* Ambrose, "Investigating student understanding in intermediate mechanics: Identifying the need for a tutorial approach to instruction," *Am. J. Phys.* **72** (2004).

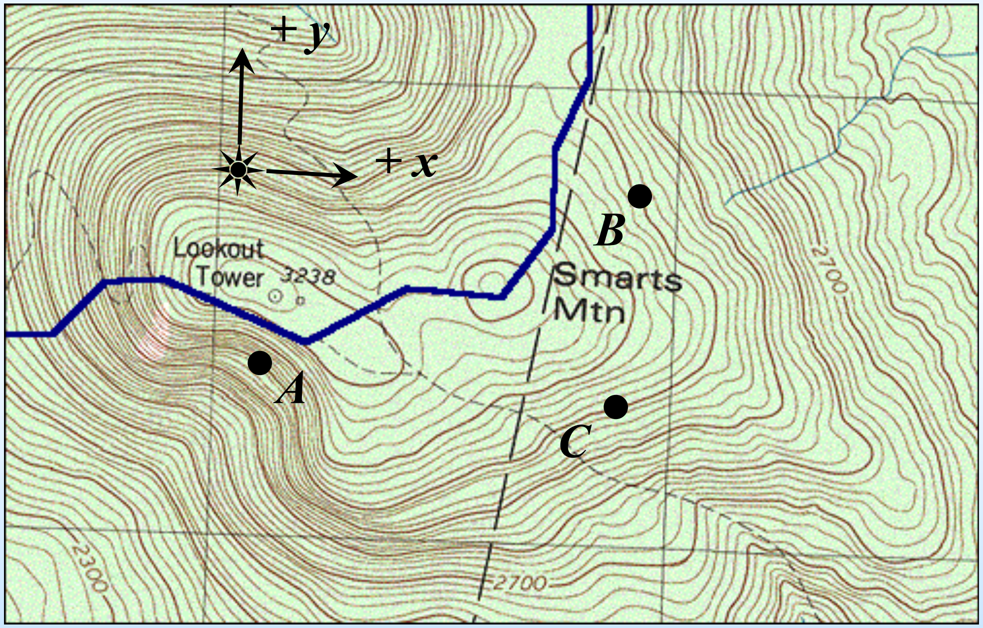
Building students' physical ***and*** mathematical intuitions about conservative forces

In the tutorial *Conservative forces and equipotential diagrams:*

Students develop a qualitative relationship between **force vectors** and local **equipotential contours**…

…and construct an **operational definition of the gradient** of potential energy:

$$\nabla U = \left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} \right)$$

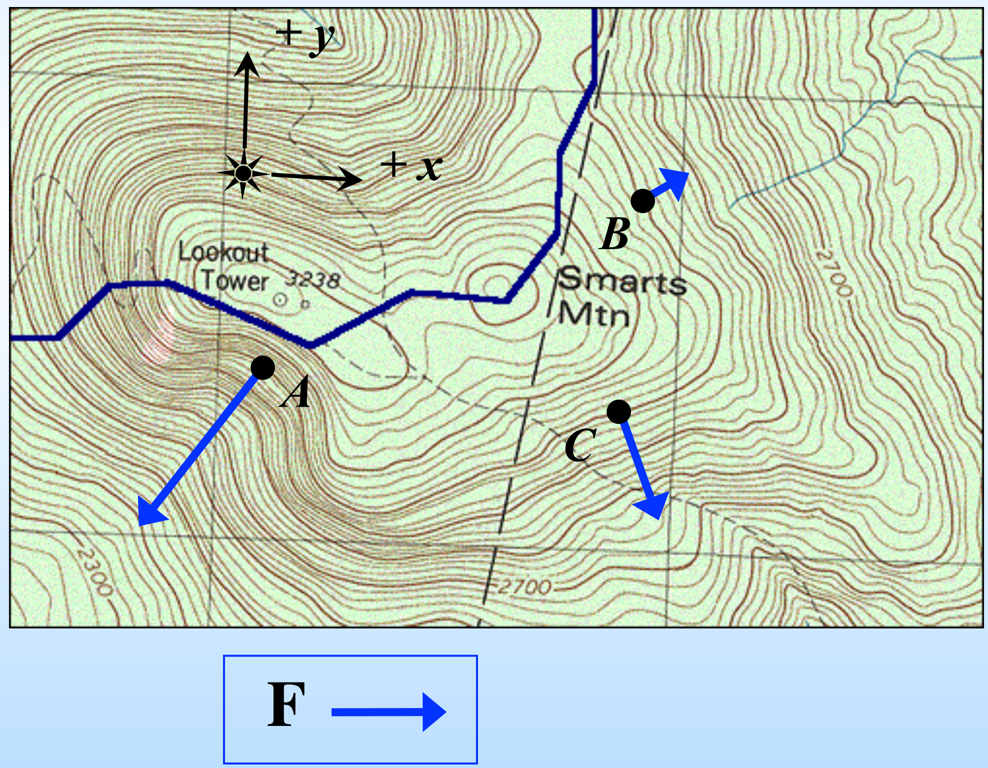


Tutorial: *Conservative forces and equipotential diagrams*

Topographic maps as analogues to equipotential diagrams

For the three labeled locations *(A – C),* students:

- Rank locations according to ***slope***
- Rank locations according to ***net force*** (neglecting friction)
- Determine ***direction*** of net force

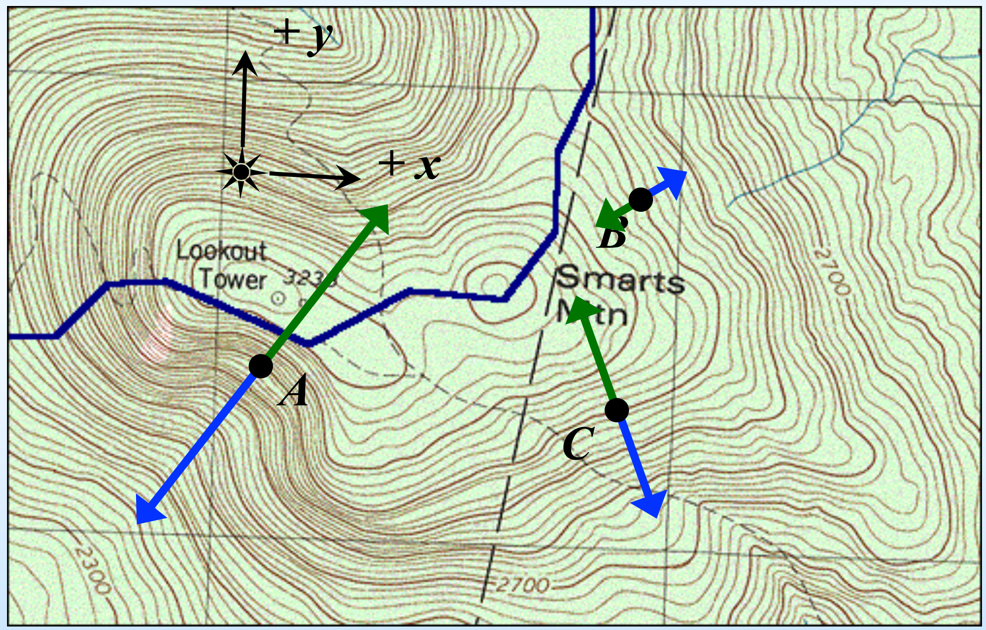


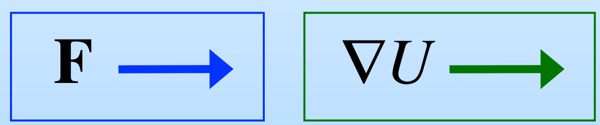
Tutorial: ***Conservative forces and equipotential diagrams***

Operational definition of *gradient*

For the three labeled locations *(A – C),* students answer:

- *In words,* how would you calculate $\frac{\partial U}{\partial x}$ and $\frac{\partial U}{\partial y}$?
- Is $\frac{\partial U}{\partial x}$ pos, neg, or zero?
- Is $\frac{\partial U}{\partial y}$ pos, neg, or zero?
- Compare $\left|\frac{\partial U}{\partial x}\right|$ and $\left|\frac{\partial U}{\partial y}\right|$.
- Draw $\nabla U = \left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j}\right)$.





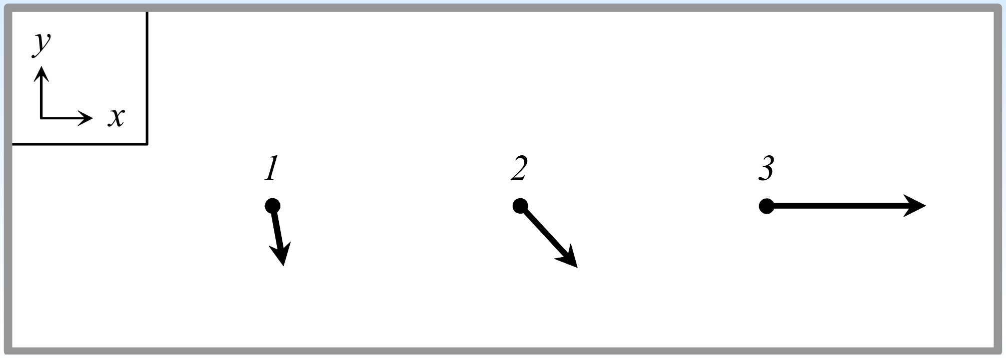
“Unknown equipotentials” post-test

Exam after tutorial, GVSU, 2003 ($N = 7$)

Three identical particles are located at the labeled locations *(1, 2, and 3)*.

Each vector represents the force **F***(x, y)* exerted at that location, with:

$$F_3 > F_2 > F_1$$



A. In the space above, *carefully sketch an equipotential diagram* for the region shown. Make sure your equipotential lines are consistent with the force vectors shown. Explain the reasoning you used to make your sketch.

B. On the basis of your results in part A, rank the labeled locations according to the *potential energy* of the particle at that location. Explain how you can tell.

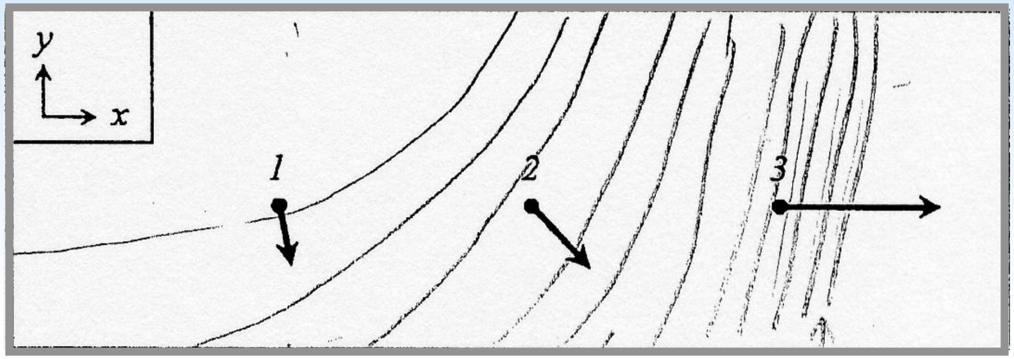
"Unknown equipotentials" post-test: Results

Exam after tutorial, GVSU, 2003 ($N = 7$)

Three identical particles are located at the labeled locations *(1, 2, and 3)*.

Each vector represents the force $\mathbf{F}(x, y)$ exerted at that location, with:

$$F_3 > F_2 > F_1$$



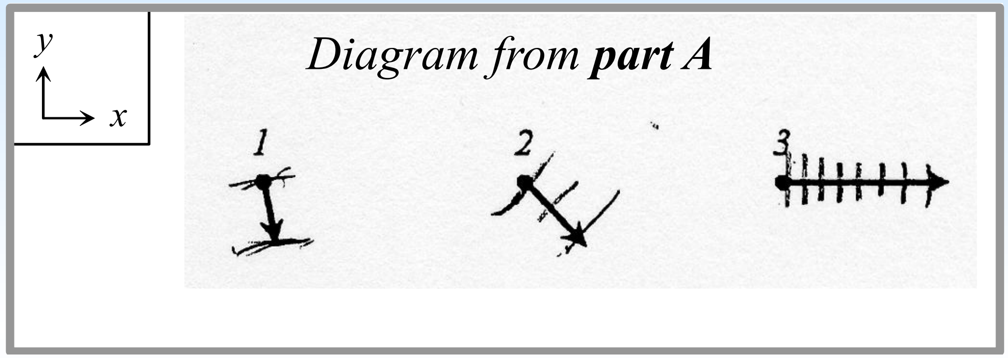
Acceptable student diagram *(part A)*

Part A:	Relative spacing of equipotentials:	**4/7 correct**
	Orientation of equipotentials:	**5/7 correct**
Part B:	Rank points by potential energy:	**1/7 correct**

"Unknown equipotentials" post-test: Results

Exam after tutorial, GVSU, 2003 ($N = 7$)

Example of a partially correct response:



Part B *(rank points by potential energy):* 3 > 2 > 1

The greater the force, the higher potential energy $\vec{F} = -\nabla V$

Persistent *confusion between a quantity (potential energy U) and its rate of change (force* $\mathbf{F} = -\nabla U$*)*

Helping students understand what the gradient *means* <u>and</u> what it *does not mean*

Last page of tutorial includes these questions:

Summarize your results: Does the vector representing ∇U ...

- point in the direction of *increasing* or *decreasing* potential energy?
- point in the direction in which potential energy changes the *most* or the *least* with respect to position?
- ▶ have the *same magnitude* at all locations having the *same potential energy?* Explain why or why not.

Role of tutorial homework

Students apply and extend results from tutorial

Students reflect upon what "∇U" means and what it does not mean:

Consider the following statement:

"For a conservative force, the magnitude of the force is related to potential energy. The larger the potential energy, the larger the magnitude of the force."

Do you *agree* or *disagree* with this statement?

- If you agree, state so explicitly. Explain your reasoning.
- If you disagree, use your results from this tutorial to provide **at least three (3)** specific counterexamples. Explain your reasoning.

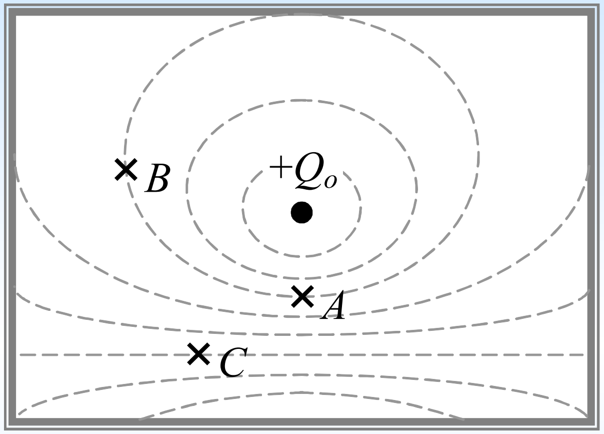
Examples of assessment questions

On written exams after tutorial instruction

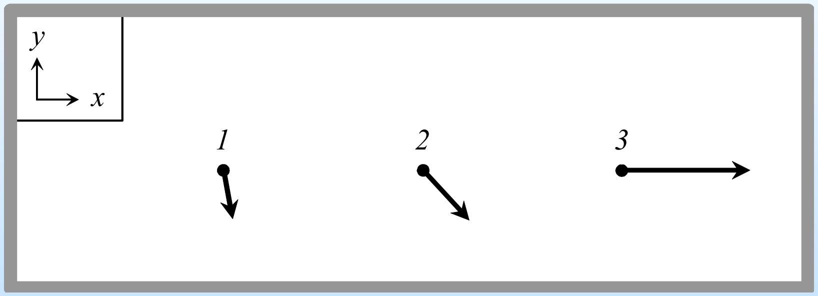
Task: Given equipotential map, predict directions and relative magnitudes of forces.

GVSU: **20/23 correct** (2 classes)

SPU: **8/11 correct** (1 class)



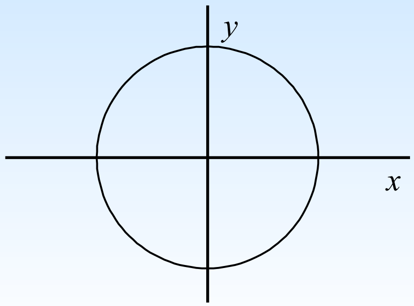
Task: Given several forces, sketch a possible equipotential map and rank points by potential energy.



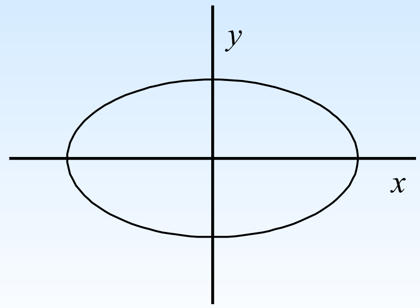
GVSU: **14/30 correct** (3 classes)

Tutorial: ***Harmonic motion in two dimensions***

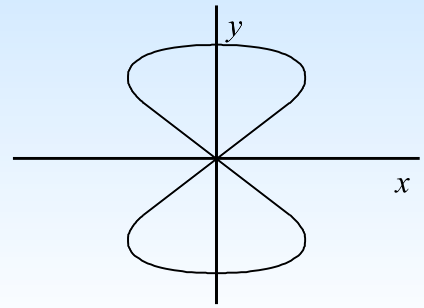
Connecting force constants and frequencies



Case #1



Case #2



Case #3

Students are guided to recognize:

- how many oscillations occur along the *y*-axis for each oscillation along the *x*-axis
- how differences in force constants affect periods and frequencies
- how phase difference between *x*- and *y*-motions affect trajectories of isotropic oscillators

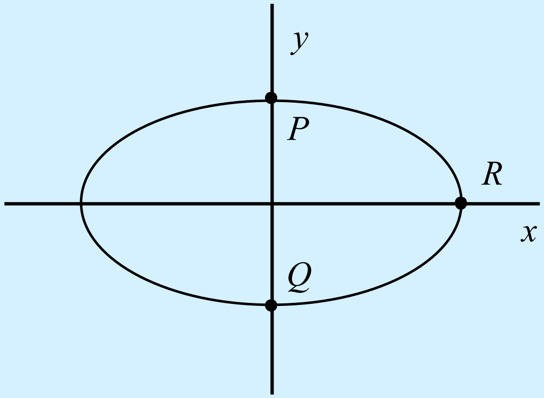
Tutorial homework: ***Harmonic motion in two dimensions***

Connecting amplitude and potential energy (<u>not</u> frequency)

Excerpt from tutorial homework:

A. Critique the following statement. Explain.

> "The oscillator goes farther in the *x*-direction than in the *y*-direction. That means the spring in the *y*-direction must be stiffer than the spring in the *x*-direction."

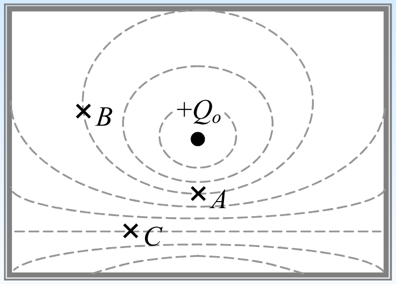


B. Rank points *P, Q,* and *R* according to (i) total energy, (ii) potential energy, (iii) kinetic energy.

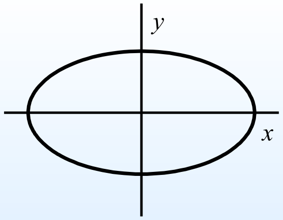
Explain how the difference in the *x*- and *y*-amplitudes, used *incorrectly* in the statement in part A, can help justify a *correct* answer here in part B.

Summary: Even physics majors…

- Experience difficulty discriminating between a quantity and its rate of change.
  - Force and potential energy: $\mathbf{F} = -\nabla U$



- Use incomplete or inappropriate "compensation arguments"
  - Relationship between amplitude and frequency



- Need guidance to connect mathematics and physics
  - Differential operators (gradient, curl)
  - Graphical representations (motion graphs, trajectory plots)

Preliminary conclusions

- Intermediate mechanics students often experience conceptual and reasoning difficulties similar to those identified at the introductory level.
  - Deeply-seated alternate conceptions
  - Disparate "knowledge pieces"*
- Specific difficulties must be addressed explicitly and repeatedly for meaningful learning.
- An inquiry-based tutorial approach can be effective for both investigating and improving student understanding.

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* Scherr, Am. J. Phys. **75** (2007).

Intermediate Mechanics Tutorials

Project website: **http://perlnet.umaine.edu/IMT**

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