Using research to enhance student learning in intermediate mechanics

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Outline of presentation

- Introduction and motivation for project
- Investigating the nature of student thinking: How do students extract physical meaning from the mathematics?
- Using research to design and assess a tutorial approach for teaching intermediate mechanics
  - *Example #1:* Conservative forces
  - *Example #2:* Central forces and angular momentum
- Summary and reflections

From previous research at the introductory level

After standard lecture instruction in introductory physics, most students:*

- lack a *functional understanding* of many basic physical concepts

  (*i.e.,* they lack the ability to apply a concept in a context different from that in which the concept was introduced)

- lack a coherent framework relating those concepts

* McDermott and Redish, "Resource letter PER-1: Physics Education Research," Am. J. Phys. **67** (1999).

What is “intermediate mechanics” about?

Review of fundamental topics

- Vectors
- Kinematics
- Newton’s laws
- Work, energy, energy conservation
- Linear and angular momentum

New applications and extensions

- Velocity-dependent forces
- Linear and non-linear oscillations
- Conservative force fields
- Non-inertial reference frames
- Central forces, Kepler’s laws

New formalism and representations

- Scalar and vector fields; del operator; gradient, curl
- Phase space diagrams

Research questions in the teaching of intermediate mechanics

- When conceptual or reasoning difficulties arise, do student ideas tend to be:
  - robust and deeply-seated?
  - based on naïve intuitions?

- Which instructional strategies seem to be productive in addressing such difficulties, and under what circumstances?
  - *elicit-confront-resolve*[1]
  - building and/or refining intuitions[2]

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[1] McDermott, *Am. J. Phys.* **61** (1993), 295 – 298.
[2] Elby, *Am. J. Phys.* Phys. Ed. Res. Suppl. **69** (2001), S54 – S64.

Intermediate Mechanics Tutorials

Collaboration between GVSU (Ambrose)* and U. Maine (M. Wittmann)

- Simple harmonic motion
- Newton's laws and velocity-dependent forces
- Damped harmonic motion
- Driven harmonic motion
- Phase space diagrams
- Conservative force fields
- Harmonic motion in two dimensions
- Accelerating reference frames
- Orbital mechanics
- Generalized coordinates and Lagrangian mechanics

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* Ambrose, "Investigating student understanding in intermediate mechanics: Identifying the need for a tutorial approach to instruction," *Am. J. Phys.* **72** (2004).

Context of investigation and curriculum development

Primary student populations: Intermediate mechanics

- Grand Valley State University (GVSU)
- University of Maine (U. Maine)
- Seattle Pacific University (SPU)
- Pilot sites for *Intermediate Mechanics Tutorials*

Primary research methods

- Ungraded quizzes (pretests)
- Written examinations

"Explain your reasoning."

- Formal and informal observations in classroom
- Individual and group student clinical interviews

Example #1

Conservative forces

What we teach about conservative forces

in intermediate mechanics

A force $\vec{F}(\vec{r})$ is conservative if and only if:

- the work by that force around any closed path is zero
- $\vec{\nabla} \times \vec{F} = 0$ at all locations
- a potential energy function $U(\vec{r})$ exists so that $\vec{F} = -\vec{\nabla} U$

(generalization of $\vec{E} = -\vec{\nabla} V$ from electrostatics)

Research question: What difficulties do students have in understanding and applying this relationship?

From previous research at the introductory level

Many students have difficulty discriminating between a **quantity** and its **rate of change:**

- position *vs.* velocity*
- velocity *vs.* acceleration *
- height *vs.* slope of a graph **
- electric field *vs.* electric potential †
- electric (or magnetic) flux *vs.* change in flux
- *…and many other examples*

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* Trowbridge and McDermott, Am. J. Phys. **48** (1980) and **49** (1981); Shaffer and McDermott, Am. J. Phys. **73** (2005).

** McDermott, Rosenquist, and van Zee, Am. J. Phys. **55** (1987).

† Allain, Ph.D. dissertation, NCSU, 2001; Maloney *et al.,* Am. J. Phys. Suppl. **69** (2001).

“Equipotential map” pretest

Intermediate mechanics

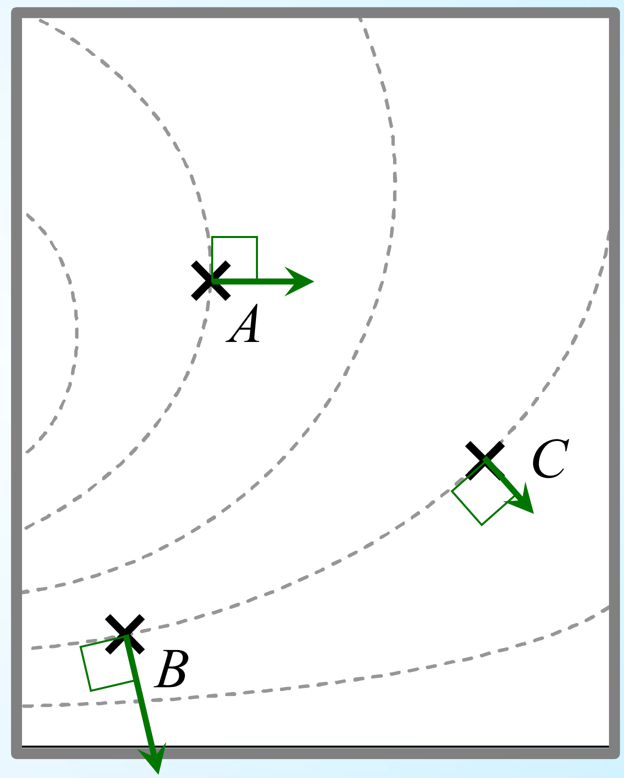
After all lecture instruction in introductory E&M

In the region of space depicted at right, the dashed curves indicate locations of *equal potential energy* for a test charge $+q_{test}$ placed within this region.

It is known that the potential energy at location *A* is *greater than* that at *B* and *C*.

A. At each location, draw an arrow to indicate the direction in which the test charge $+q_{test}$ would move when released from that location. Explain.

B. Rank the locations *A, B,* and *C* according to the magnitude of the force exerted on the test charge $+q_{test}$. Explain your reasoning.



(Qualitatively correct force vectors are shown.)

Equipotential map pretest: Results

Intermediate mechanics, GVSU (N = 73, 8 classes)

After all lecture instruction in introductory E&M

Percent correct *with correct reasoning:*
(rounded to nearest 5%)

Part A (Directions of force vectors)	50%	(35/73)
Part B (Ranking force magnitudes)	20%	(14/73)
Both parts correct	**15%**	**(9/73)**

Similar results have been found among students at U. Maine and pilot test sites.

Equipotential map pretest: Results

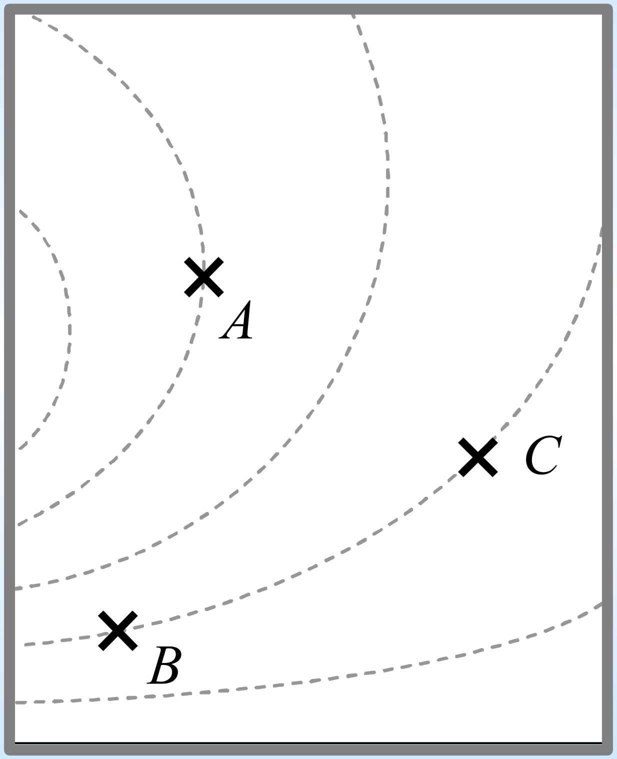
Intermediate mechanics

After all lecture instruction in introductory E&M

Most common *incorrect* ranking: $F_A > F_B = F_C$

Example: "A has the highest potential so it can exert a larger force on a test charge. B and C are on the same potential curve and thus have equal abilities to exert force."

Example: "A has the most potential pushing the charge fastest. B & C are on the same level."



Failure to discriminate between a quantity *(potential energy U)* ***and its rate of change*** *(force $\vec{F} = -\vec{\nabla} U$)*

Equipotential map pretest: Results

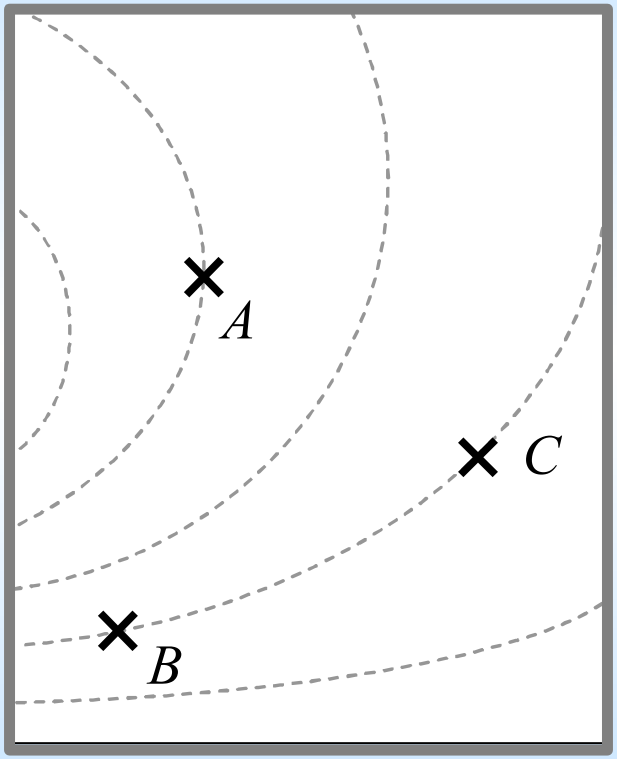
Intermediate mechanics

After all lecture instruction in introductory E&M

Most common *incorrect* ranking: $F_A > F_B = F_C$

Example: "Since F is proportional to V, higher V means higher F."

Example:
"[$V_A > V_B = V_C$] … $F(x) = -dV/dx$
$\therefore$ $F_C = F_B$ in magnitude and
$F_A > F_C$ in magnitude."



Failure to discriminate between a quantity *(potential energy U)* ***and its rate of change*** *(force $\vec{F} = -\vec{\nabla} U$)*

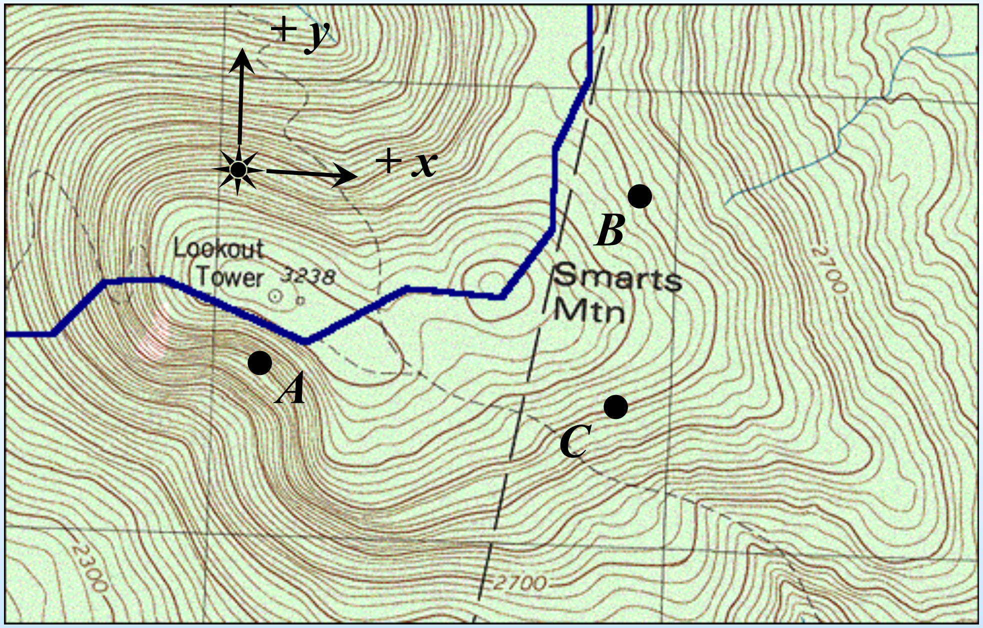
Building students' physical ***and*** mathematical intuitions about conservative forces

In the tutorial *Conservative forces and equipotential diagrams:*

Students develop a qualitative relationship between **force vectors** and local **equipotential contours**…

…and construct an **operational definition of the gradient** of potential energy:

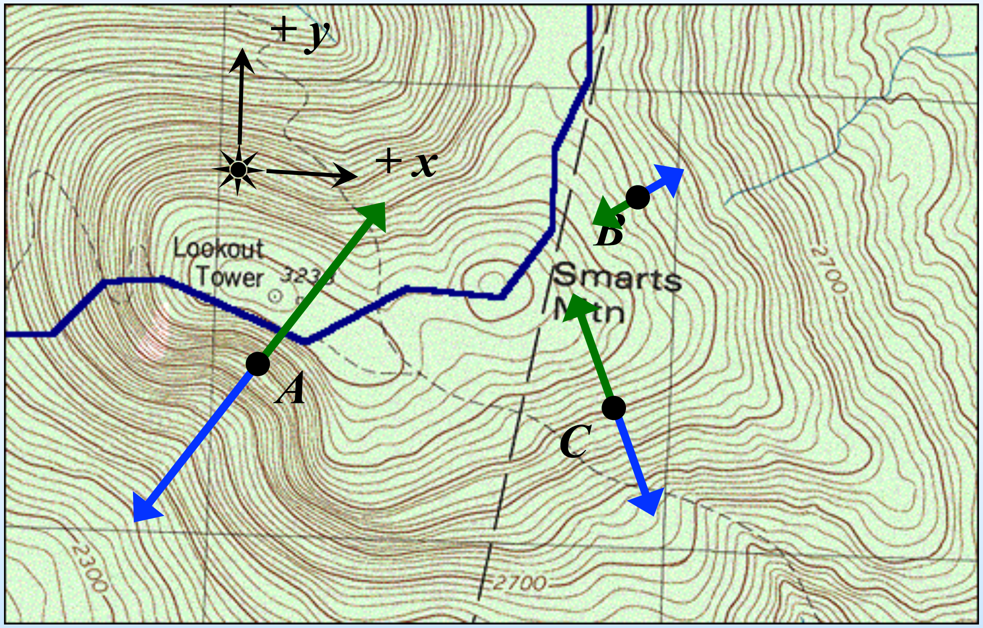
$$\vec{\nabla}U = \left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} \right)$$

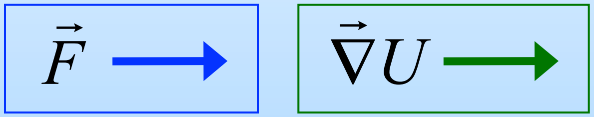


Building students' physical *and* mathematical intuitions about conservative forces

For the three labeled locations *(A – C),* students answer:

- *In words,* how would you calculate $\frac{\partial U}{\partial x}$ and $\frac{\partial U}{\partial y}$?
- Is $\frac{\partial U}{\partial x}$ pos, neg, or zero?
- Is $\frac{\partial U}{\partial y}$ pos, neg, or zero?
- Compare $\left|\frac{\partial U}{\partial x}\right|$ and $\left|\frac{\partial U}{\partial y}\right|$.
- Draw $\vec{\nabla} U = \left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j}\right)$.





Students reflect upon what gradient *means* <u>and</u> what it *does not mean*

Last page of tutorial includes these questions:

Summarize your results: Does $\vec{\nabla} U$ ...

- point in the direction of *increasing* or *decreasing* potential energy?
- point in the direction in which potential energy changes the *most* or the *least* with respect to position?
- have the *same magnitude* at all locations having the *same potential energy?* Explain why or why not.

Students reflect upon what gradient *means* <u>and</u> what it *does not mean*

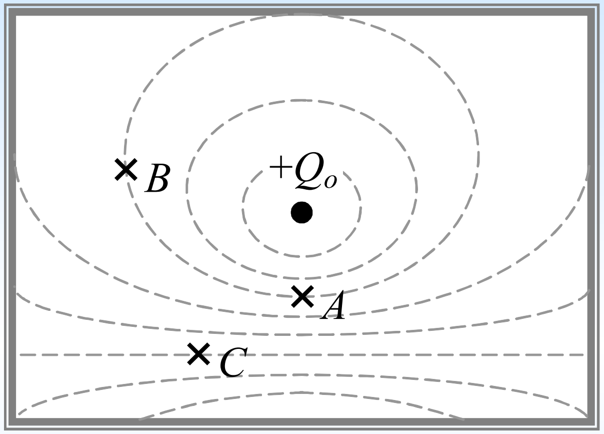
Tutorial homework includes this problem:

> Consider the following statement:
>
> "For a conservative force, the magnitude of the force is related to potential energy. The larger the potential energy, the larger the magnitude of the force."
>
> Do you *agree* or *disagree* with this statement?
>
> – If you agree, state so explicitly. Explain your reasoning.
>
> – If you disagree, use your results from this tutorial to provide <u>**at least three (3)**</u> specific counterexamples. Explain your reasoning.

Examples of assessment questions

On written exams after tutorial instruction

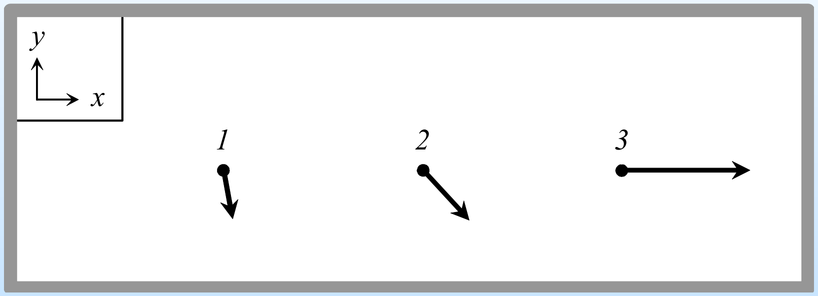
Task: Given equipotential map, predict directions and relative magnitudes of forces.



GVSU: **20/23 correct** (2 classes)

SPU: **8/11 correct** (1 class)

Task: Given several forces, sketch a possible equipotential map and rank points by potential energy.



GVSU: **14/30 correct** (3 classes)

Example #2

Central forces and angular momentum

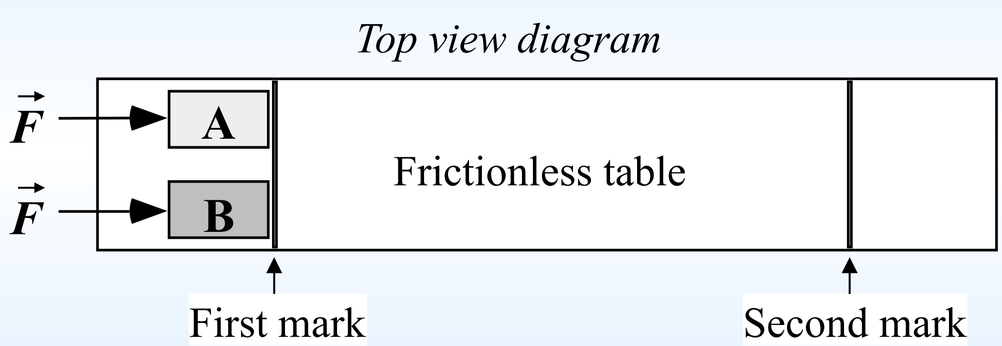
Research question: What difficulties do students have in understanding that changes in angular momentum are caused by a net torque?

From previous research at the introductory level

Students use inappropriate **“compensation arguments”** when comparing quantities that involve two or more variables.

Example: Two carts, $m_A < m_B$, are at rest on a level, frictionless table.

Equal forces are exerted on the carts as they move between the two marks.*



Students often incorrectly predict:

“$\boldsymbol{KE_A > KE_B}$” because faster speed of A “matters more” than mass ($KE = \frac{1}{2}mv^2$)

“$\boldsymbol{p_A = p_B}$” because larger mass of B “compensates for” smaller speed ($p = mv$)

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* R.A. Lawson and L.C. McDermott, *Am. J. Phys.* **55**, 811-817 (1987).

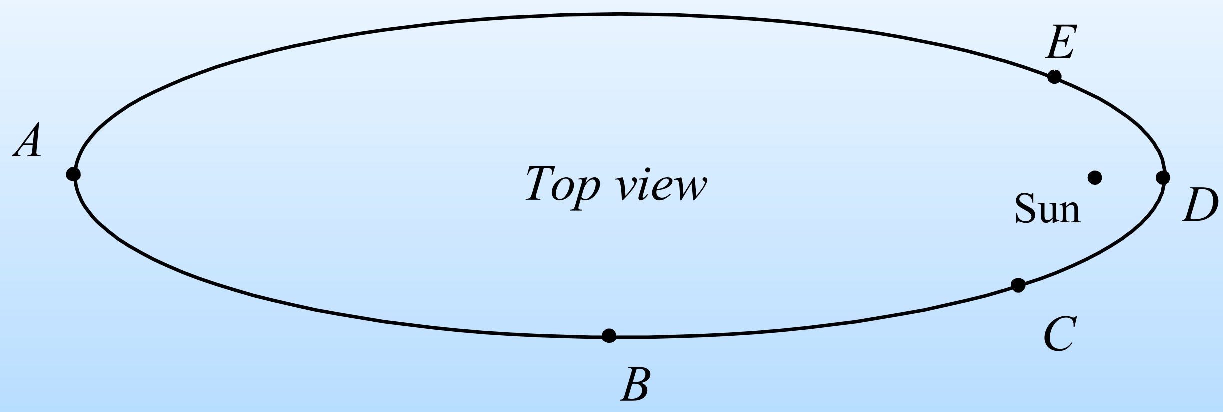
“Orbiting comet” pretest

GVSU (2 classes), WCUP (2 classes), UNH (1 class), and SPU (1 class)

A comet orbits the sun in the orbit shown below. (*Note:* This diagram is *not* a strobe photograph of the motion.)

Q1: At each point *A* – *D* is the comet moving with *increasing speed, decreasing speed,* or *constant speed?* Explain.

Q2: Rank points *A* – *D* according to the angular momentum of the comet (measured relative to the center of the Sun) at those points. Explain.

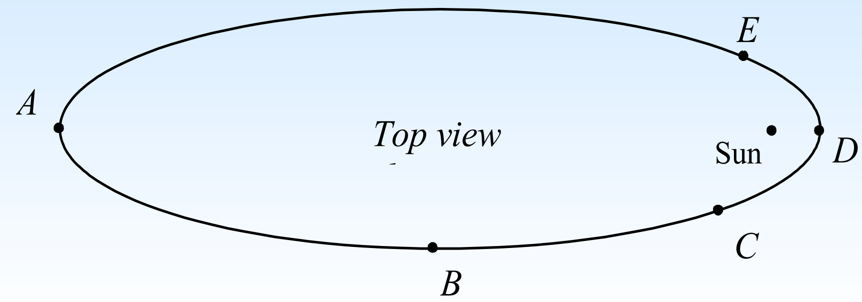


Orbiting comet pretest: Results

GVSU (2 classes), WCUP (2 classes), UNH (1 class), and SPU (1 class)

- After lecture instruction, **35% - 50%** gave **correct** rankings, *but*…

- Most correct rankings were based on incomplete "compensation arguments" or incorrect reasoning.



Example of a correct response based on incorrect reasoning:

"All have the same angular momentum…because gravity is a conservative force."

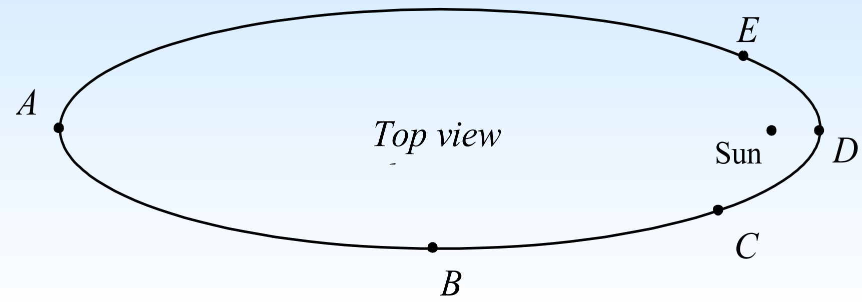
Example of a correct response based on "compensation argument":

"All equal since if v is increasing, r is decreasing, and vice versa."

Orbiting comet pretest: Results

GVSU (2 classes), WCUP (2 classes), UNH (1 class), and SPU (1 class)

- Many incorrect rankings were based solely on a single variable:



*Example of incorrect ranking based solely on **speed**:*

"D > E = C > B > A. …since as the object moves closer to the sun v increases, so momentum increases."

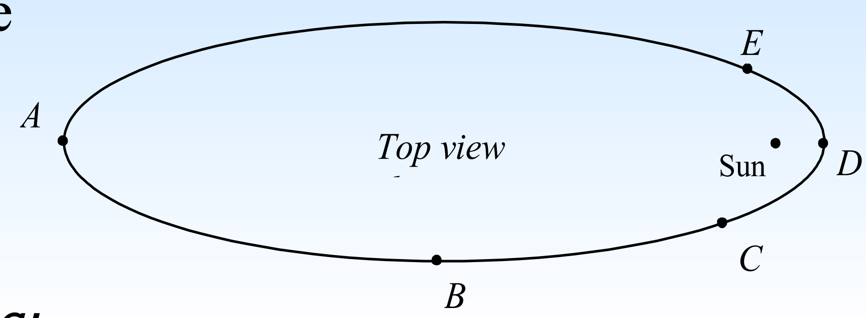
*Example of incorrect ranking based solely on **radius of curvature**:*

"A, D > C > B. A & D must have the most angular momentum to execute the tightest turn."

Orbiting comet pretest: Results

GVSU (2 classes), WCUP (2 classes), UNH (1 class), and SPU (1 class)

- Many incorrect or incomplete rankings were based on incorrect intuitions:



Example of incomplete ranking:

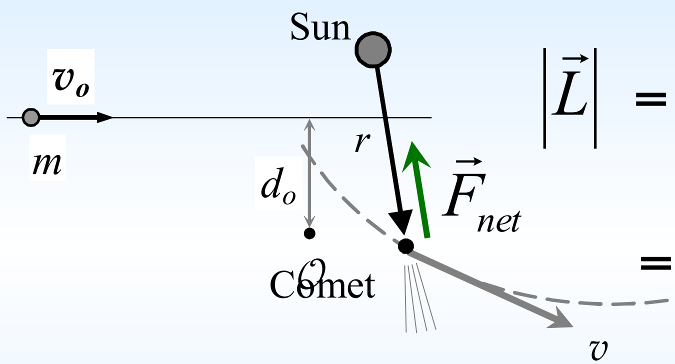
"*v*"	"*cos* θ"	"*r*"
D	D, A	A
E	E	B
C	C	C
B	B	E
A		D

"I am not sure. Too many things countering others."

Building students' physical *and* mathematical intuitions about angular momentum

In the tutorial *Angular momentum and Kepler's second law:*

Students gain practice applying definition of **angular momentum**…



$$\left|\vec{L}\right| = \left|\vec{r} \times m\vec{v}\right| = \begin{cases} \boxed{mr_\perp v} \\ mrv_\perp \end{cases}$$

…and are guided to construct a test for **angular momentum conservation:**

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}\left(\vec{r} \times m\vec{v}\right) = \cancel{\left(\frac{d\vec{r}}{dt} \times m\vec{v}\right)} + \boxed{\left(\vec{r} \times m\frac{d\vec{v}}{dt}\right)}$$

Torque by net force
(point particle regime)

Examples of assessment questions

On exams after tutorial instruction, GVSU (1 class)

Task #1:

Imagine that we lived in a world in which Newton's law of gravitation was an inverse *cubic* law:

$$\vec{F}(\vec{r}) = -\frac{kMm}{r^3}\hat{e}_r$$

A. Show that the above force is conservative. Explain and/or show all work.

B. Do objects moving under the influence of the above force conserve angular momentum (relative to the origin)? Explain and/or show all work.

~ 60% (9/15) correct with correct explanations

Ex.: "[Yes.] Both force and position have only $\hat{e}_r$ components, thus they act along the same line, so $\vec{r} \times \vec{F} = 0$ and $d\vec{L} / dt = 0$."

Examples of assessment questions

On exams after tutorial instruction, GVSU (1 class) and SPU (1 class)

Task #2:

A small plastic puck is placed on a frictionless horizontal air table. The puck is made to move under the influence of a (net) force (where c is a constant):

$$\vec{F}(x,y) = -xy\hat{i} - cx^2\hat{j}$$

A. For what value of c is the force conservative? Show all work. (*Ans.:* $c = \frac{1}{2}$)

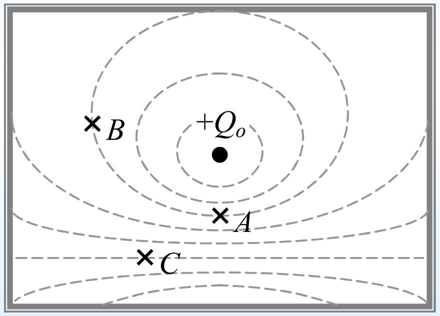
B. As the puck moves under the influence of this force, would its angular momentum (with respect to the origin) be conserved? Explain and/or show all work.

Most common incorrect reasoning (~ 30%)

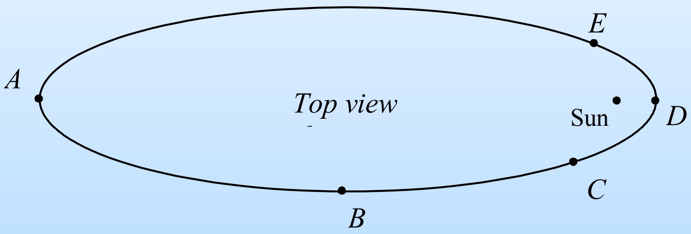
"Angular momentum is conserved because the curl $F = 0$ and so the force is conservative."

Summary and reflections

- Physics majors in *advanced* courses can and do experience conceptual and reasoning difficulties similar to those already identified at the *introductory* level.
  - Difficulty discriminating between a **quantity** and its **rate of change**



  - Reliance on inappropriate **"compensation arguments"** with two or more variables rather than cause-effect relationships



Summary and reflections

- Specific difficulties must be addressed *explicitly* and *repeatedly* for meaningful learning to occur.

- Students need guidance to extract physical meaning from the mathematics.
  - **Guided sense-making** seems more important than derivations.
  - Students need practice articulating **in their own words** the meaning of the mathematics.

Intermediate Mechanics Tutorials

Project website: **http://perlnet.umaine.edu/IMT**

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