Using research to improve learning in upper division physics courses: An example in intermediate mechanics

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Supported by NSF grants DUE-0441426 and DUE-0442388

Special acknowledgements

- Michael Wittmann *(University of Maine)*

- Lillian C. McDermott, Peter Shaffer, Paula Heron
  (University of Washington)

- Stamatis Vokos, John Lindberg *(Seattle Pacific University),*
  Dawn Meredith *(University of New Hampshire),*
  Carolyn Sealfon *(West Chester University of Pennsylvania),*
  Carrie Swift *(University of Michigan-Dearborn),*
  Juliet Brosing *(Pacific University)*

- National Science Foundation

Outline of presentation

- Context and motivation for investigation
  - Some lessons learned from physics education research at introductory level
- Research to *probe student thinking* in intermediate mechanics:
  - *Example #1:* Conservative forces
  - *Example #2:* Harmonic motion in 1D and 2D
- Research to *enhance student learning* in intermediate mechanics
- Summary and discussion

Context of investigation and curriculum development

Primary student populations: Intermediate mechanics

- Grand Valley State University (GVSU)
- University of Maine (U. Maine)
- Seattle Pacific University (SPU)
- Pilot sites for *Intermediate Mechanics Tutorials* project

Primary research methods

- Ungraded quizzes (pretests)
- Written examinations

"Explain your reasoning."

- Formal and informal observations in classroom
- Individual and group student clinical interviews

Challenges in physics education research

(especially with advanced topics)

What we measure: student responses

What we infer: student reasoning, knowledge structure, beliefs, expectations

Given a research instrument or task, it may be difficult to tell whether students proceed:

- from reasoning → to response
- from response → to justification

Common topics in intermediate mechanics

Review of fundamental topics

- Vectors
- Kinematics
- Newton's laws
- Work, energy, energy conservation
- Linear and angular momentum

New applications and extensions

- Velocity-dependent forces
- Linear and non-linear oscillations
- Conservative force fields
- Non-inertial reference frames
- Central forces, Kepler's laws

New formalism and representations

- Scalar and vector fields;
  del operator; gradient, curl
- Phase space diagrams

From previous research at the introductory level

After standard lecture instruction in introductory physics, most students:*

– lack a *functional understanding* of many basic physical concepts

(*i.e.,* they lack the ability to apply a concept in a context different from that in which the concept was introduced)

– lack a coherent framework relating those concepts

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* McDermott and Redish, "Resource letter PER-1: Physics Education Research," Am. J. Phys. **67** (1999).

From previous research at the introductory level

Many students have difficulty discriminating between a **quantity** and its **rate of change:**

- position *vs.* velocity*
- velocity *vs.* acceleration*
- height *vs.* slope of a graph**
- electric field *vs.* electric potential †
- electric (or magnetic) flux *vs.* change in flux
- *…and many other examples*

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* Trowbridge and McDermott, Am. J. Phys. **48** (1980) and **49** (1981); Flores and Kanim, Am. J. Phys. **72** (2004); Shaffer and McDermott, Am. J. Phys. **73** (2005).

** McDermott, Rosenquist, and van Zee, Am. J. Phys. **55** (1987).

† Allain, Ph.D. dissertation, NCSU, 2001; Maloney *et al.,* Am. J. Phys. Suppl. **69** (2001).

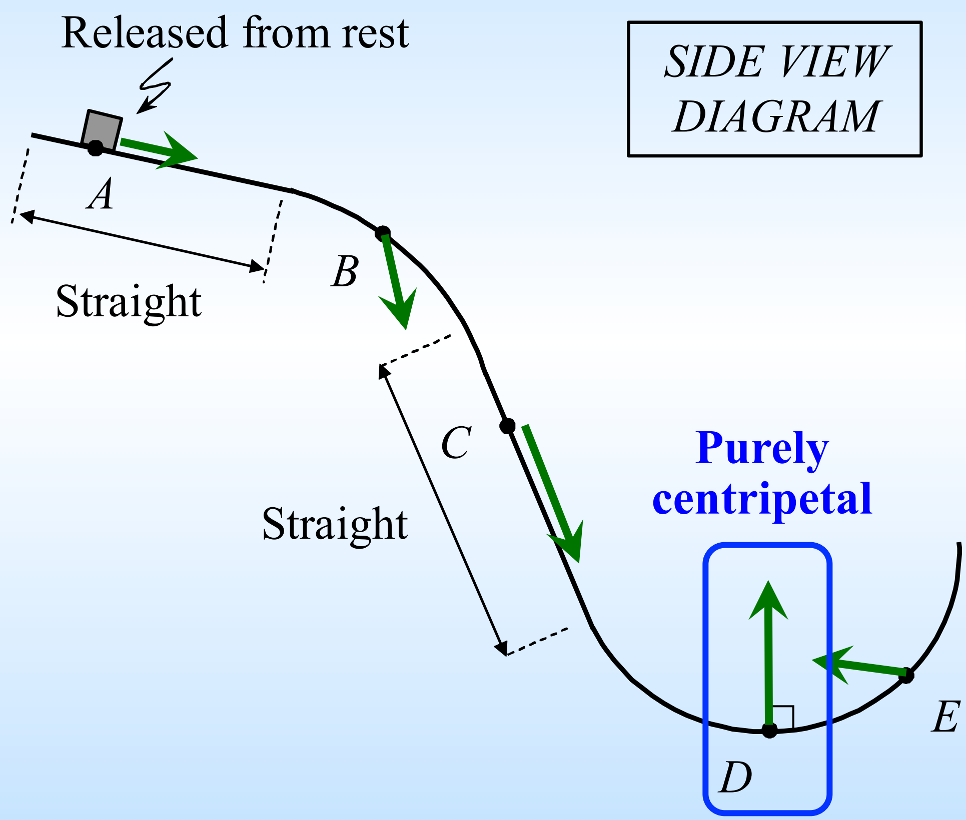
"Curved ramp" task (2D kinematics)

Intermediate mechanics, GVSU

A long, frictionless ramp, consisting of straight and curved portions, is shown. A block is released from rest at point *A*.

At each labeled point, draw arrows to indicate the directions of the (i) ***velocity*** and (ii) ***acceleration*** of the block at that point.

If the acceleration is zero at any point, indicate so explicitly.

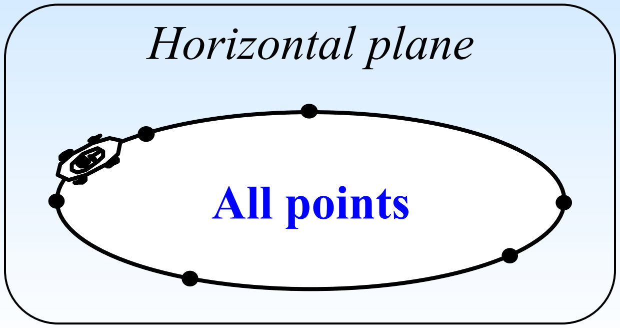


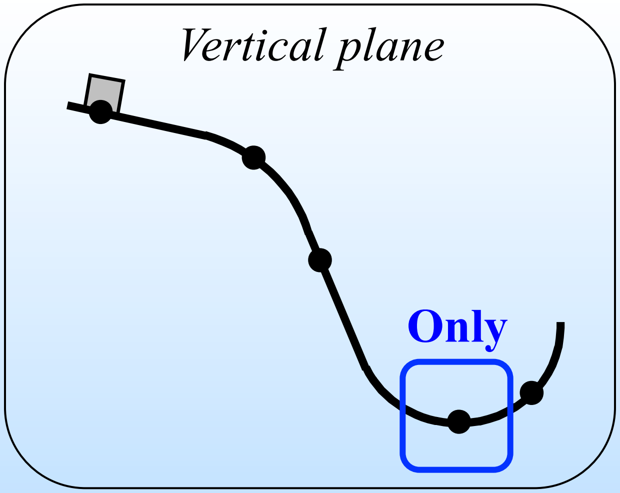
Results from questions after traditional instruction

Acceleration in a horizontal *or* vertical plane

Centripetal cases only

Correct responses with correct reasoning *(to nearest 5%)*	
Algebra-based mechanics, GVSU ($N \sim 200$)	**5%**
Calculus-based mechanics,* UW ($N \sim 7{,}000$)	**20%**
Intermediate mechanics, GVSU ($N = 22$)	**25%**
Physics graduate students,* UW ($N \sim 75$)	**65%**





* Shaffer and McDermott, Am. J. Phys. **73** (2005).

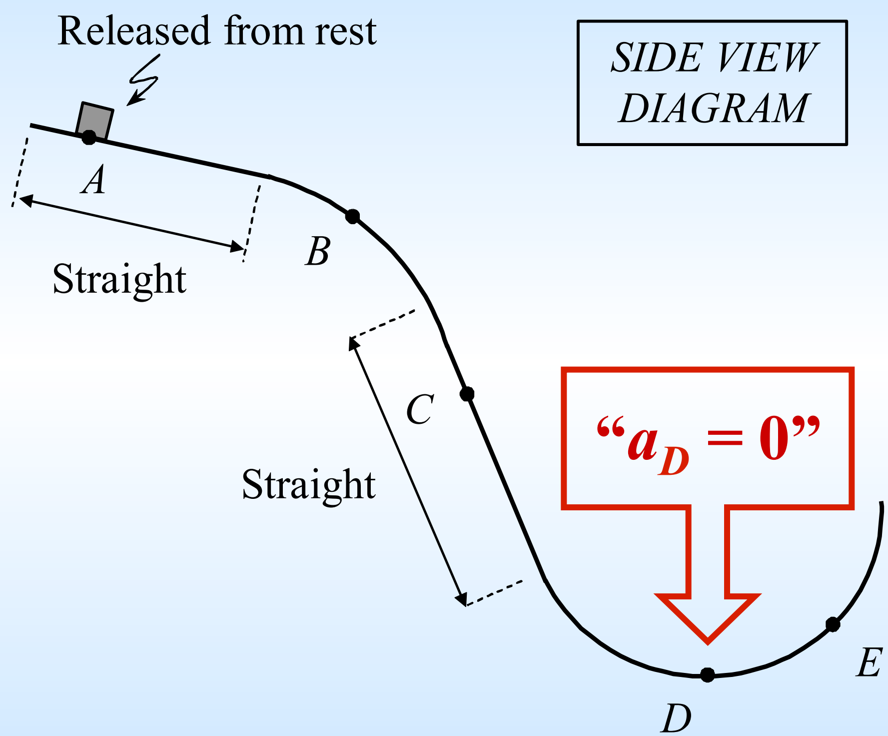
Curved ramp pretest: Student difficulties

Intermediate mechanics, GVSU

Most students incorrectly stated that acceleration is ***zero*** at point D

Example:

"The block is not falling [at point *D*]."



Failure to distinguish between a quantity *(vertical velocity)* ***and its rate of change*** *(vertical acceleration)*

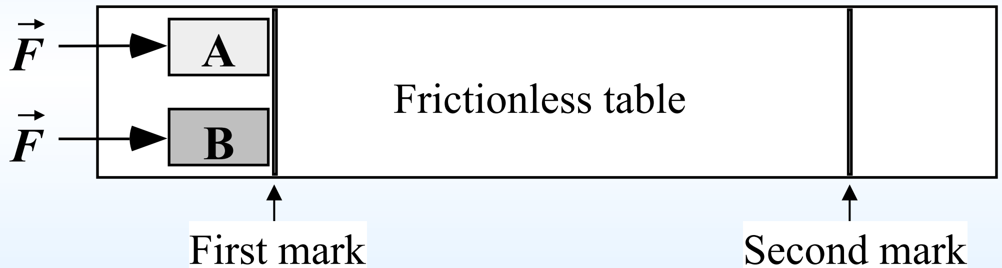
From previous research at the introductory level

Students use inappropriate **"compensation arguments"** when comparing quantities that involve two or more variables.

Example: Two carts, $m_A < m_B$, are at rest on a level, frictionless table.

Equal forces are exerted on the carts as they move between the two marks.*

Top view diagram



After instruction, introductory physics students often incorrectly predict:

"$KE_A > KE_B$" because faster speed of A "matters more" than mass ($KE = \frac{1}{2}mv^2$)

"$p_A = p_B$" because larger mass of B "compensates for" smaller speed ($p = mv$)

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* R.A. Lawson and L.C. McDermott, *Am. J. Phys.* **55** (1987); O'Brien Pride, Vokos, and McDermott, *Am. J. Phys.* **66** (1998).

Research questions from teaching and learning of intermediate mechanics

- What prevalent conceptual and reasoning difficulties arise, and to what extent are they:
  - rooted in basic concepts?
  - intrinsic to more advanced material?
  - based on connections between physics and mathematics?
  - characterized by deeply entrenched ideas or naïve intuitions? [1]
- Which instructional strategies seem to be productive in addressing such difficulties, and under what circumstances?
  - *elicit-confront-resolve*[2]
  - building and/or refining intuitions[3]

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[1] Scherr, Am. J. Phys. **75** (2007).
[2] McDermott, *Am. J. Phys.* **61** (1993), 295 – 298.
[3] Elby, *Am. J. Phys.* Phys. Ed. Res. Suppl. **69** (2001), S54 – S64

Example #1: Conservative forces and potential energy

A force $\vec{F}(\vec{r})$ is conservative if and only if:

- the work by that force around any closed path is zero
- $\vec{\nabla} \times \vec{F} = 0$ at all locations
- $\boxed{\text{a potential energy function } U(\vec{r}) \text{ exists so that } \vec{F} = -\vec{\nabla} U}$

(generalization of $\vec{E} = -\vec{\nabla} V$ from electrostatics)

Research question: What difficulties do students have in understanding and applying this relationship?

"Equipotential map" pretest

Intermediate mechanics

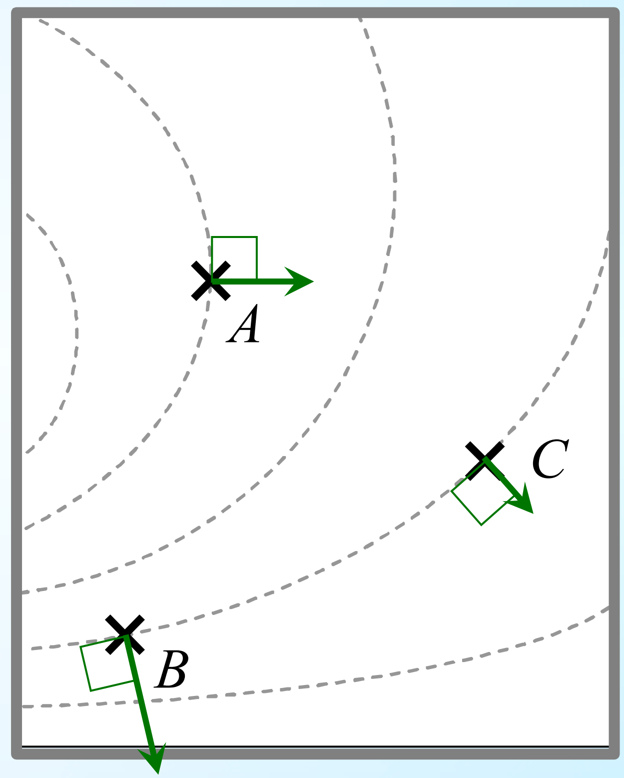
After all lecture instruction in introductory E&M

In the region of space depicted at right, the dashed curves indicate locations of *equal potential energy* for a test charge $+q_{\text{test}}$ placed within this region.

It is known that the potential energy at location *A* is *greater than* that at *B* and *C*.

A. At each location, draw an arrow to indicate the direction in which the test charge $+q_{\text{test}}$ would move when released from that location. Explain.

B. Rank the locations *A, B,* and *C* according to the magnitude of the force exerted on the test charge $+q_{\text{test}}$. Explain your reasoning.



(Qualitatively correct force vectors are shown.)

Equipotential map pretest: Results

Intermediate mechanics, GVSU (N = 73, 8 classes)

After all lecture instruction in introductory E&M

Percent correct *with correct reasoning:*
(rounded to nearest 5%)

Part A (Directions of force vectors)	50%	(35/73)
Part B (Ranking force magnitudes)	20%	(14/73)
Both parts correct	**15%**	**(9/73)**

Similar results have been found after lecture instruction at U. Maine and pilot test sites.

Equipotential map pretest: Results

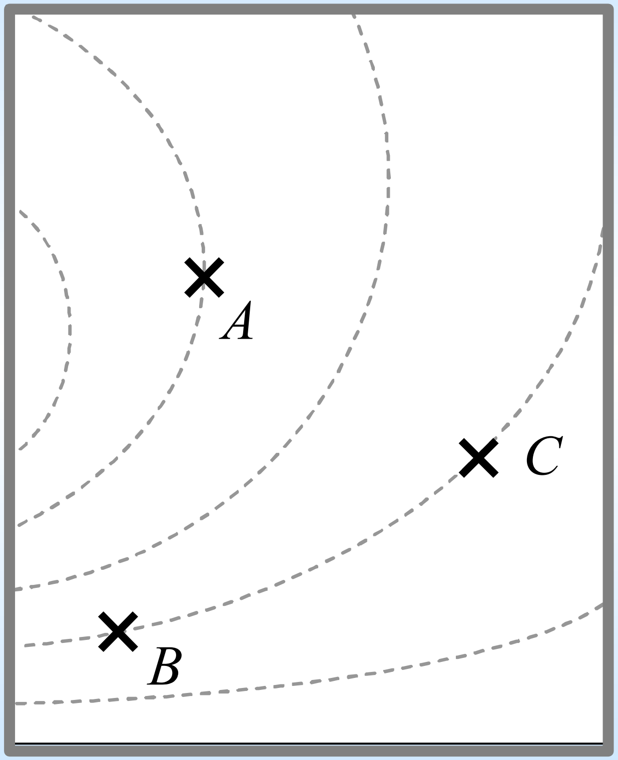
Intermediate mechanics

After all lecture instruction in introductory E&M

Most common *incorrect* ranking: $F_A > F_B = F_C$

Example: "A has the highest potential so it can exert a larger force on a test charge. B and C are on the same potential curve and thus have equal abilities to exert force."

Example: "A has the most potential pushing the charge fastest. B & C are on the same level."



Failure to discriminate between a quantity *(potential energy U)* ***and its rate of change*** *(force $\vec{F} = -\vec{\nabla} U$)*

Equipotential map pretest: Results

Intermediate mechanics

After all lecture instruction in introductory E&M

Most common *incorrect* ranking: $F_A > F_B = F_C$

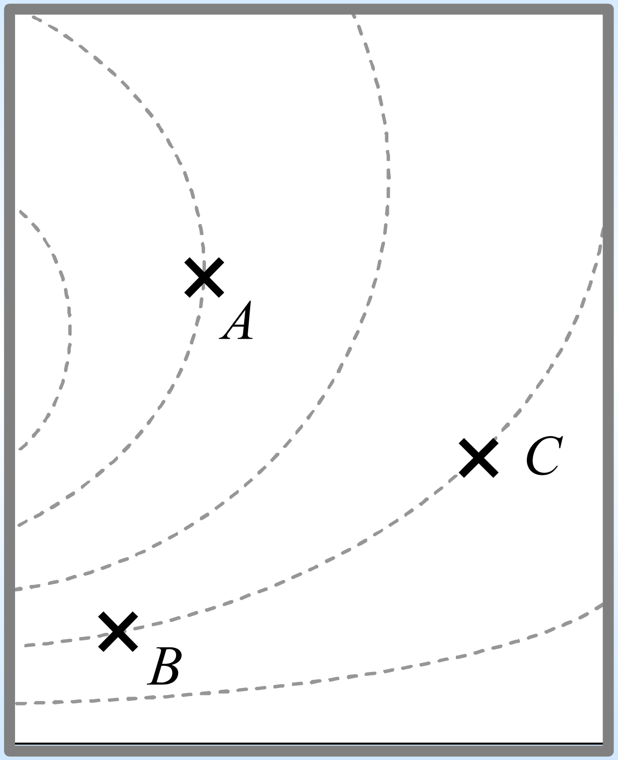
Example: "Since F is proportional to V, higher V means higher F."

Example:

"[$V_A > V_B = V_C$] … $F(x) = - dV/dx$

$\therefore$ $F_C = F_B$ in magnitude and

$F_A > F_C$ in magnitude."



Failure to discriminate between a quantity *(potential energy U)* ***and its rate of change*** *(force $\vec{F} = -\vec{\nabla} U$)*

Example #2:
Harmonic motion in one and two dimensions

	Equation of motion	Solution for *x(t)*
Simple harmonic motion	$m\ddot{x} = -kx$	$x(t) = A_o \cos(\omega_o t + \varphi)$ where $\boxed{\omega_o = \sqrt{k/m}}$
Underdamped motion $(\gamma < \omega_o)$	$m\ddot{x} = -kx - c\dot{x}$ $\left(\ddot{x} = -\omega_o^2 x - 2\gamma\dot{x}\right)$	$x(t) = A_o\, e^{-\gamma t} \cos(\omega_d t + \varphi)$ where $\omega_d = \sqrt{\omega_o^2 - \gamma^2}$

⇒ Frequency depends on mass and spring constant

⇒ Amplitude has no effect on frequency or period

Specific research questions in context of harmonic oscillations

After relevant instruction:

- How well do students understand the factors that affect the frequency of harmonic oscillations?
  - Simple harmonic motion in 1-D and 2-D
  - Damped harmonic motion
- How well do students use and interpret formal representations of oscillatory motion?
  - Motion graphs of 1D oscillators
  - Phase space diagrams of 1D oscillators
  - Real space *(x-y)* trajectories of 2D oscillators

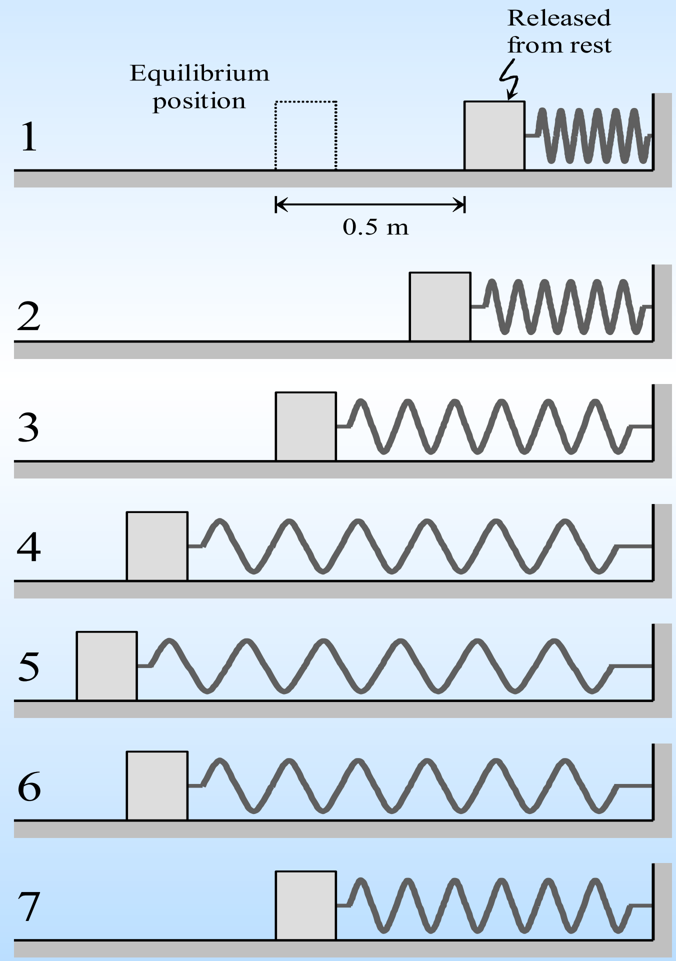
“Simple harmonic oscillator” pretest

(excerpt)

A block is connected to a spring and placed on a frictionless surface. A student releases the block <u>0.5 m</u> to the <u>right</u> of equilibrium.

For each change listed below, how (if at all) would that change affect the **period** of motion? Explain your reasoning.

- The block is released <u>0.7 m</u> to the <u>left</u> of equilibrium.
- The spring is replaced with a stiffer spring.
- The block is replaced with another block four times the mass as the original one.



Predicting effect on oscillation frequency

After lecture instruction (GVSU, 6 classes, $N \sim 50$)

The good news…

Parts ii & iii (changing ***spring*** or ***mass***):

Most students **(~ 65%)** gave qualitatively correct answers with acceptable explanations.

The bad news…

Part i (increasing ***amplitude***):

Most students answered correctly (~ 65%) but very few gave acceptable explanations.

Most common incorrect response **(~25%):**
"Larger [period if amplitude is larger], because the block travels farther during each period."

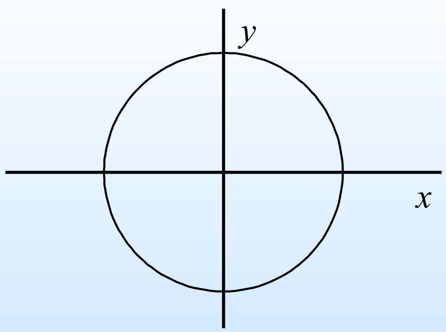
2D oscillator pretest

Consider the motion of a 2D oscillator, with $U(x, y) = ½\, k_1x^2 + ½\, k_2y^2$, or equivalently, $U(x, y) = ½\, m\omega_1^{\,2}x^2 + ½\, m\omega_2^{\,2}y^2$.

Q: For each x-y trajectory shown, could the oscillator follow that trajectory?

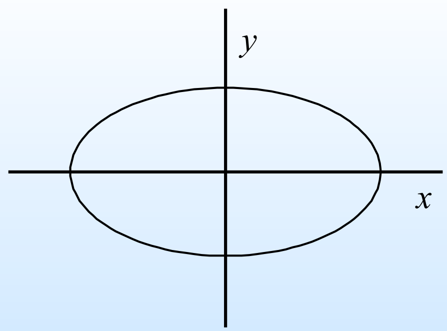
If so: Is $\boldsymbol{\omega_1}$ *greater than, less than,* or *equal to* $\boldsymbol{\omega_2}$*?* Explain.*

If not: Explain why not.



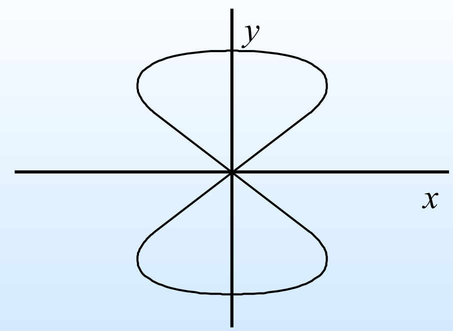
Case #1

(Ans: $\omega_1 = \omega_2$)



Case #2

(Ans: $\omega_1 = \omega_2$)



Case #3

(Ans: $\omega_1 > \omega_2$)

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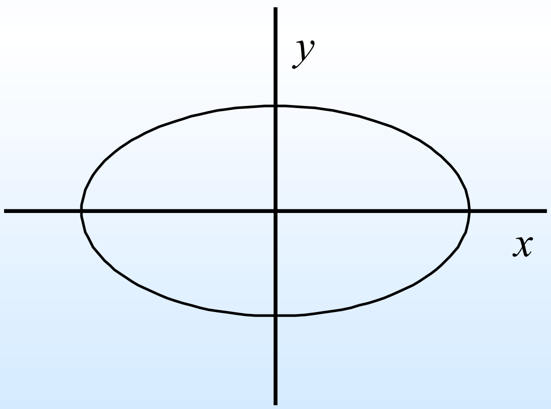
* Original phrasing asked for a comparison between $\boldsymbol{k_1}$ and $\boldsymbol{k_2}$.

2D oscillator pretest: Results

Intermediate mechanics, GVSU (4 classes) and U. Maine (1 class)

After *relevant lecture instruction*

- Few students (0% - 15%) answered all cases correctly.
- Most incorrect responses based on "compensation arguments" involving relative amplitudes along *x*- and *y*-axes:



Case #2

Example responses for Case #2:

"$k_1 < k_2$, the spring goes farther in the *x*-direction, so spring must be less stiff in that direction."

"$\omega_2 > \omega_1$. Since we now have an oval curve with the *x*-axis longer, ω_2 must be greater to compensate."

New 2D oscillator pretest

Consider an object that moves along a horizontal frictionless surface (*e.g.,* an air hockey puck on a level air table). Suppose that the object moves under the influence of a net force expressed as follows:

$$\mathbf{F}_{\mathbf{net}}(x, y) = \left(-k_x x\,\hat{i}\right) + \left(-k_y y\,\hat{j}\right)$$

Note: The above net force can be modeled by two long, mutually perpendicular springs connecting the puck to the two edges of the air table. The force constants of the springs would be k_x and k_y.

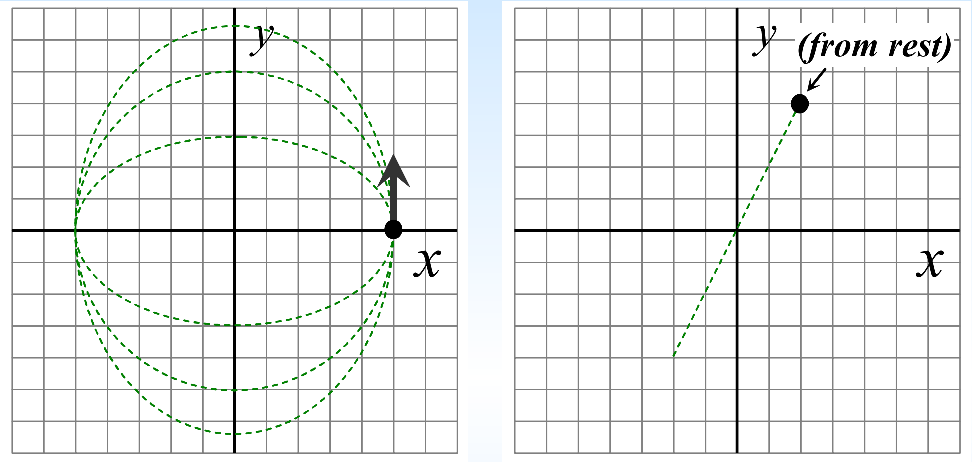
Q: For each case of the four (4) cases shown, carefully sketch a qualitatively correct *x*-*y* trajectory that the object might follow. Explain the reasoning you used to decide your answers.

New 2D oscillator pretest

(with given initial conditions and acceptable answers shown for each case)

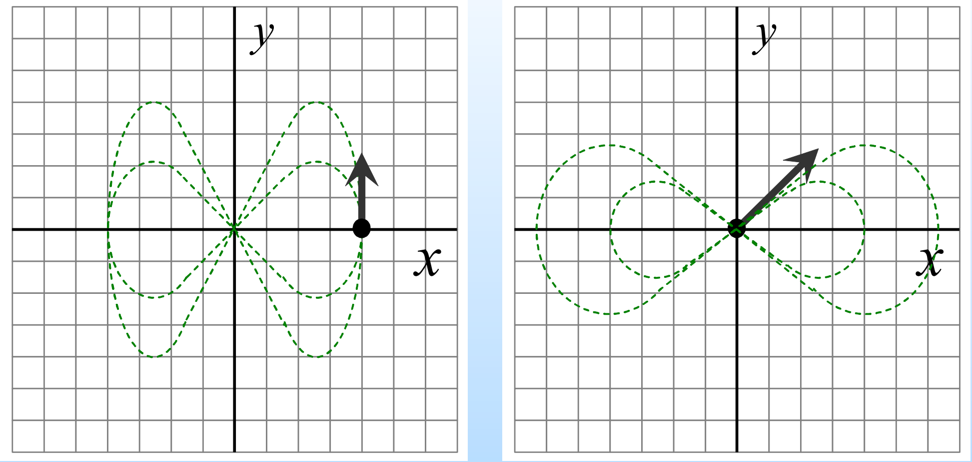
Isotropic cases:

$k_y = k_x$



Non-isotropic cases:

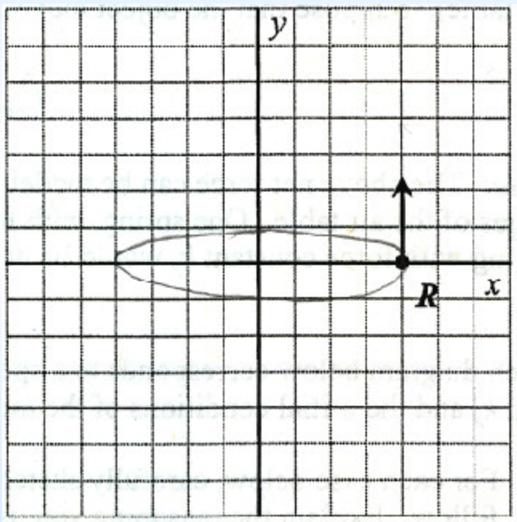
$k_y = 4k_x$



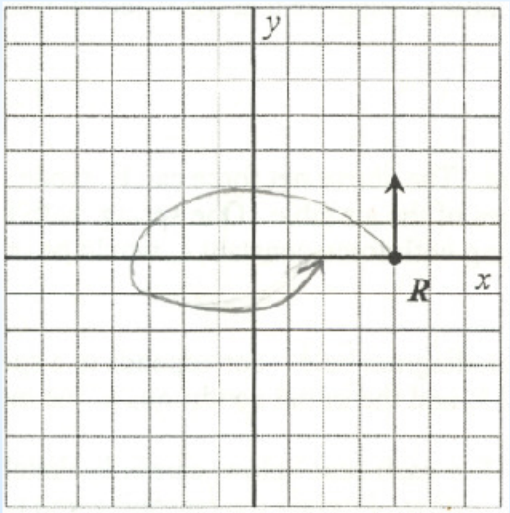
New 2D oscillator pretest: Results

GVSU (2 classes), UNH (1 class)

- "Compensation arguments" regarding amplitudes and force constants:



$k_y = 4k_x$



$k_y = 4k_x$

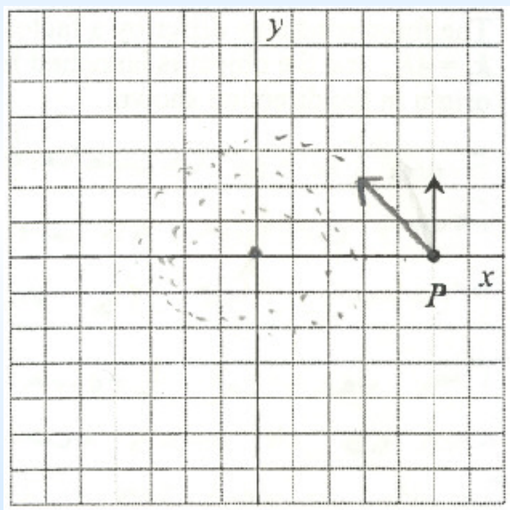
S1: "An ellipse rather than a circle because the spring forces are different."

S2: "The object travels less in the *y*-direction because of the stiffer spring. The springs attempt to return the object to equilibrium."

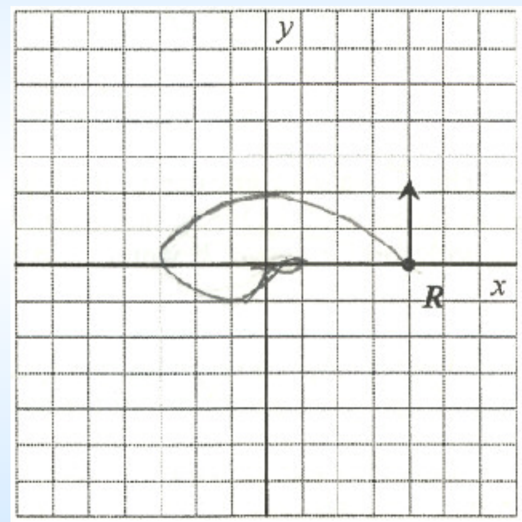
New 2D oscillator pretest: Results

GVSU (2 classes), UNH (1 class)

- Intuition that "springs pull object toward equilibrium" evident from inwardly spiraling trajectories:



$k_y = k_x$



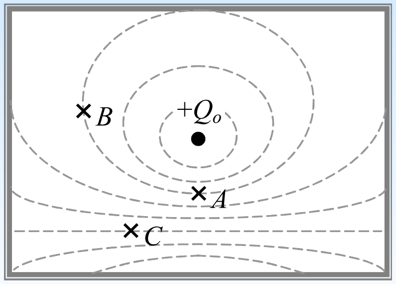
$k_y = 4k_x$

S3: "The puck wants to get back to the origin."

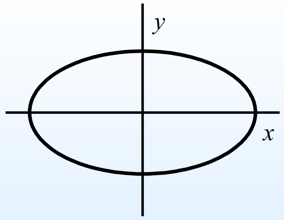
S4: "The *y*-direction will approach the origin 2 times faster than the *x*-direction and the object will eventually come to the origin."

Summary of findings: Even physics majors…

- Experience difficulty discriminating between a quantity and its rate of change.
  - Force and potential energy: $\mathbf{F} = -\nabla U$

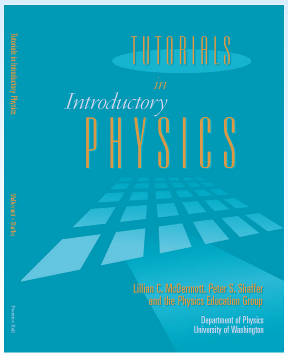


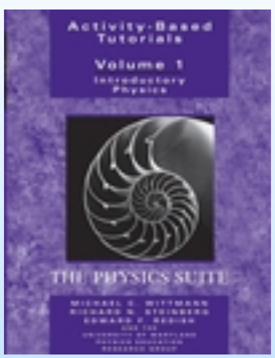
- Use incomplete or inappropriate "compensation arguments"
  - Relationship between amplitude and frequency



- Need guidance to connect mathematics and physics
  - Differential operators (gradient, curl)
  - Graphical representations (motion graphs, trajectory plots)

A research-tested guided inquiry approach for teaching *introductory* physics





- Teaching-by-questioning strategies designed to:
  - address specific conceptual and reasoning difficulties
  - help students connect the mathematics to physics
- Tutorial components:
  - pretests (ungraded quizzes, ~10 min)
  - tutorial worksheets (small-group activities, ~50 min)
  - tutorial homework
  - examination questions (post-tests)

*Intermediate Mechanics Tutorials**

Collaboration between GVSU (Ambrose) and U. Maine (M. Wittmann)

- Simple harmonic motion
- Newton's laws and velocity-dependent forces
- Damped harmonic motion
- Driven harmonic motion
- Phase space diagrams
- Conservative force fields
- Harmonic motion in two dimensions
- Accelerating reference frames
- Orbital mechanics
- Generalized coordinates and Lagrangian mechanics

* Development and dissemination support by NSF grants DUE-0441426 and DUE-0442388

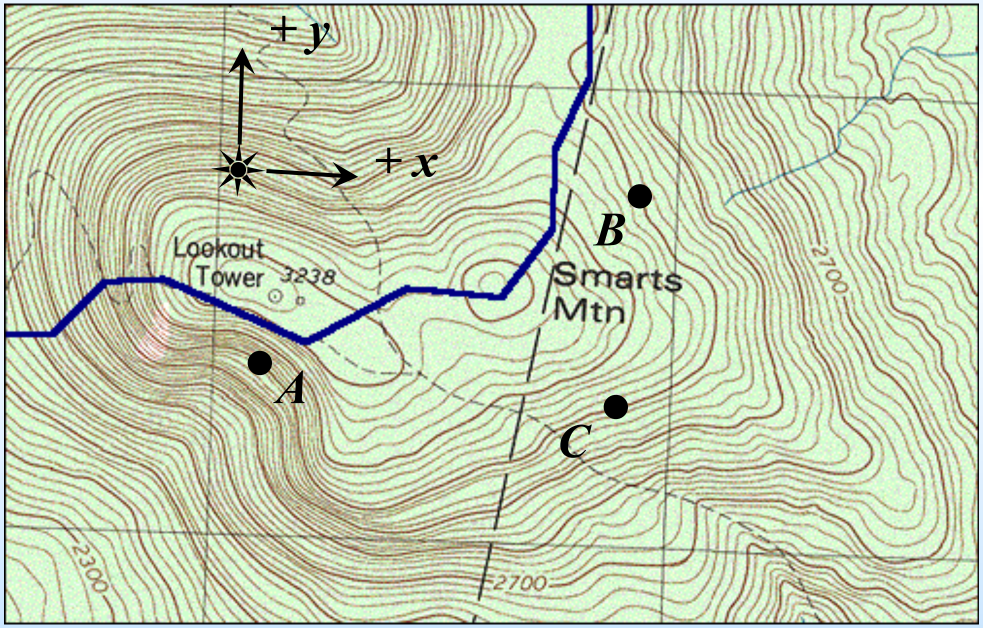
Helping students connect meaning between the physics and the mathematics

In the tutorial *Conservative forces and equipotential diagrams:*

Students develop a qualitative relationship between **force vectors** and local **equipotential contours**…

…and construct an **operational definition of the gradient** of potential energy:

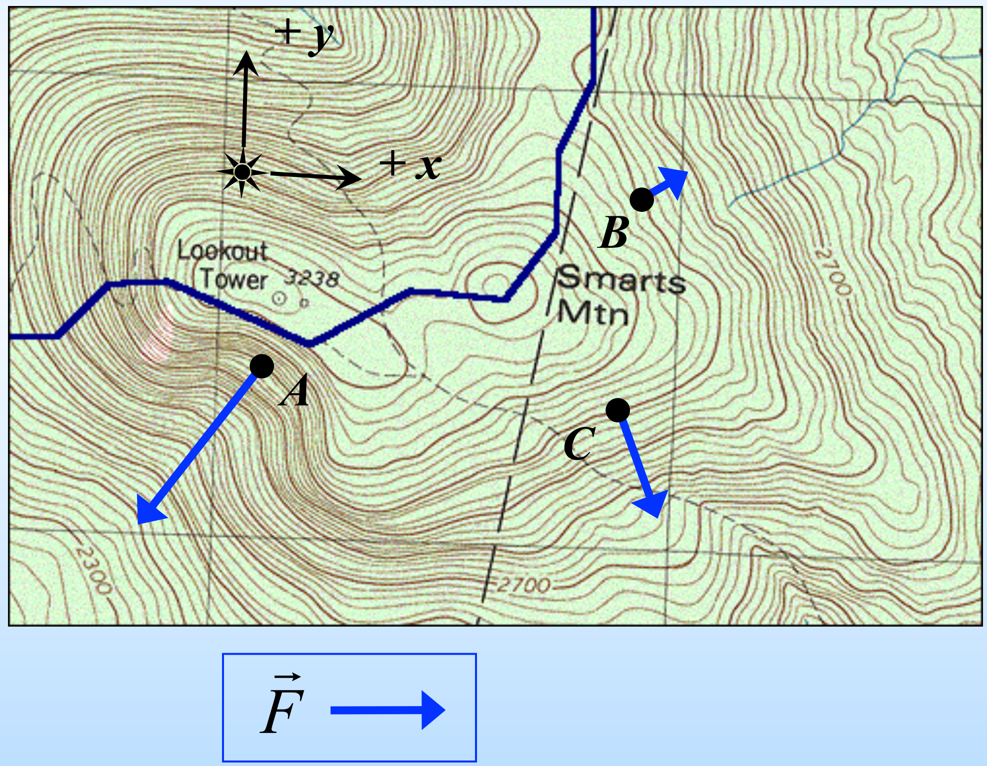
$$\vec{\nabla}U = \left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j}\right)$$



Helping students connect meaning between the physics and the mathematics

Students use topographic maps as analogues to equipotential maps:

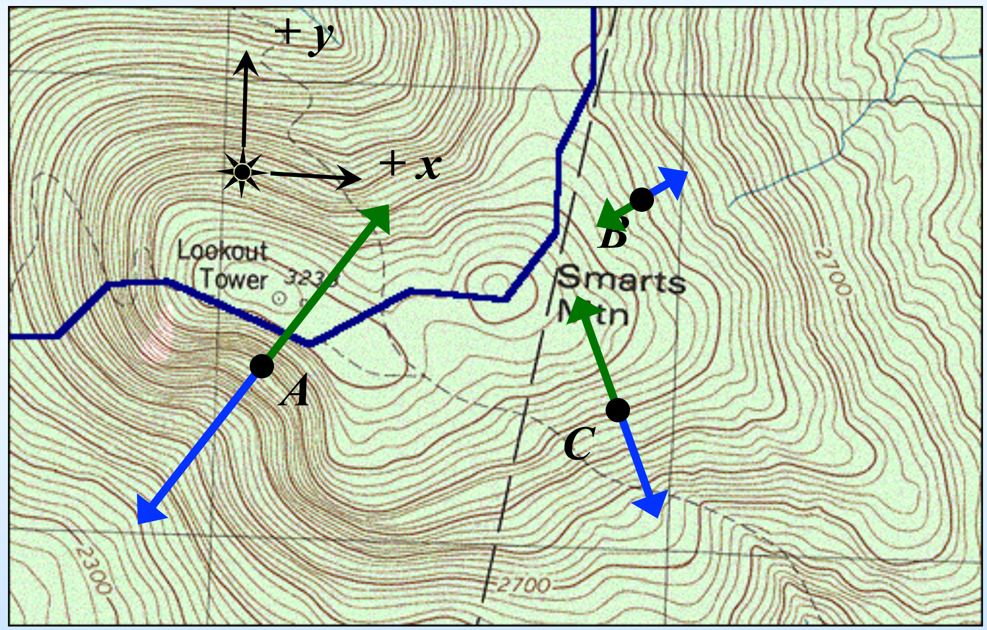
- Rank locations according to ***slope***
- Rank locations according to ***net force*** (neglecting friction)
- Determine ***direction*** of net force



Helping students connect meaning between the physics and the mathematics

Students then construct an operational definition of *gradient:*

- *In words,* how would you calculate $\dfrac{\partial U}{\partial x}$ and $\dfrac{\partial U}{\partial y}$?
- Is $\dfrac{\partial U}{\partial x}$ pos, neg, or zero?
- Is $\dfrac{\partial U}{\partial y}$ pos, neg, or zero?
- Compare $\left|\dfrac{\partial U}{\partial x}\right|$ and $\left|\dfrac{\partial U}{\partial y}\right|$.
- Draw $\vec{\nabla} U = \left(\dfrac{\partial U}{\partial x}\hat{i} + \dfrac{\partial U}{\partial y}\hat{j}\right)$.



$\vec{F} \longrightarrow$ $\vec{\nabla} U \longrightarrow$

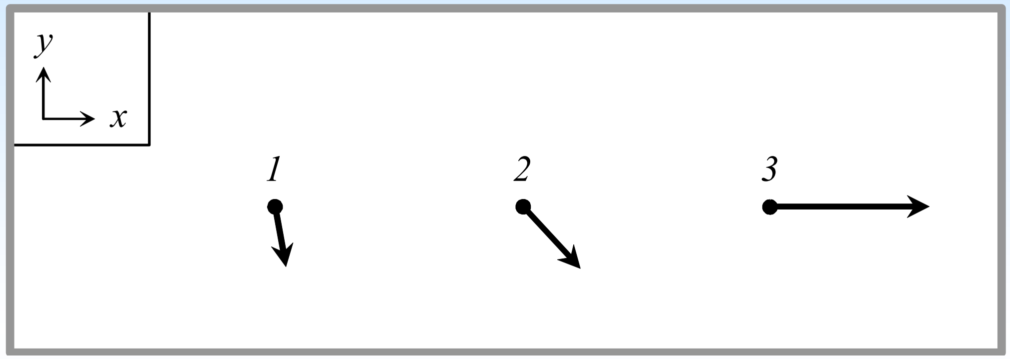
"Unknown equipotentials" post-test

Exam after tutorial, GVSU 2003 ($N = 7$)

Three identical particles are located at the labeled locations *(1, 2, and 3)*.

Each vector represents the force $\boldsymbol{F}(x, y)$ exerted at that location, with:

$$F_3 > F_2 > F_1$$



A. In the space above, *carefully sketch an equipotential diagram* for the region shown. Make sure your equipotential lines are consistent with the force vectors shown. Explain the reasoning you used to make your sketch.

B. On the basis of your results in part A, rank the labeled locations according to the *potential energy* of the particle at that location. Explain how you can tell.

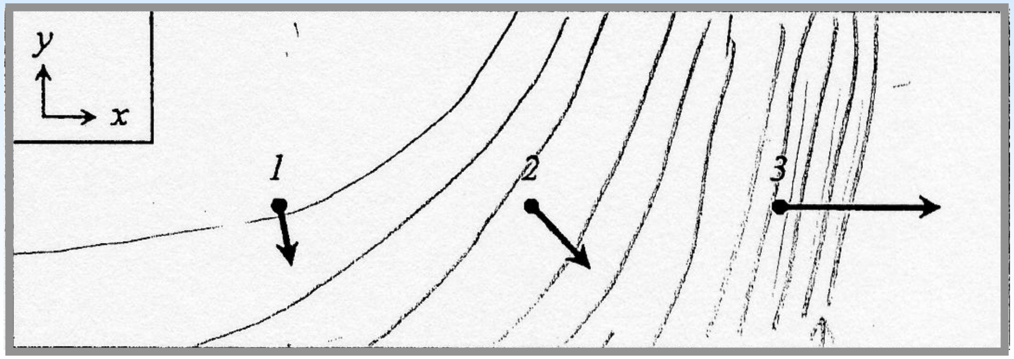
“Unknown equipotentials” post-test: Results

Exam after tutorial, GVSU 2003 ($N = 7$)

Three identical particles are located at the labeled locations *(1, 2, and 3)*.

Each vector represents the force $\boldsymbol{F}(x, y)$ exerted at that location, with:

$$F_3 > F_2 > F_1$$



Acceptable student diagram *(part A)*

Part A: Relative spacing of equipotentials: **4/7 correct**

Orientation of equipotentials: **5/7 correct**

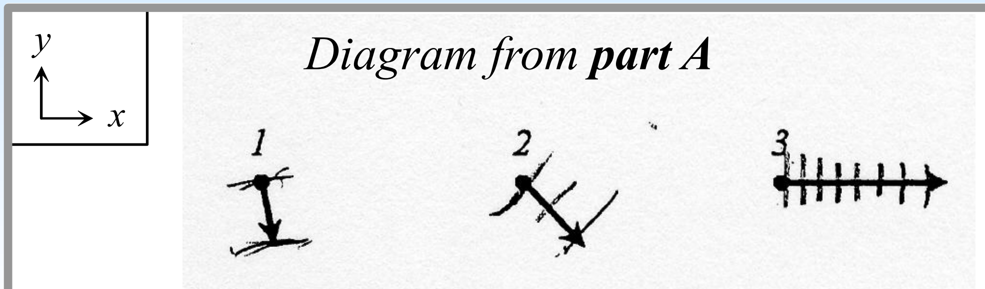
Part B: Rank points by potential energy: **1/7 correct**

"Unknown equipotentials" post-test: Results

Exam after tutorial, GVSU 2003 ($N = 7$)

Example of a partially correct response:

*Diagram from **part A***



Part B *(rank points by potential energy):* 3 > 2 > 1

The greater the force, the higher potiental energy $\vec{F} = -\nabla V$

Persistent *confusion between a quantity (potential energy U) and its rate of change (force $\vec{F} = -\vec{\nabla} U$)*

Students reflect upon what gradient *means* <u>and</u> what it *does not mean*

Tutorial activity concludes with these questions:

Summarize your results: Does $\vec{\nabla}U$ ...

- point in the direction of *increasing* or *decreasing* potential energy?
- point in the direction in which potential energy changes the *most* or the *least* with respect to position?
- have the *same magnitude* at all locations having the *same potential energy?* Explain why or why not.

Students reflect upon what gradient *means* <u>and</u> what it *does not mean*

Tutorial homework includes this problem:

Consider the following statement:

"For a conservative force, the magnitude of the force is related to potential energy. The larger the potential energy, the larger the magnitude of the force."

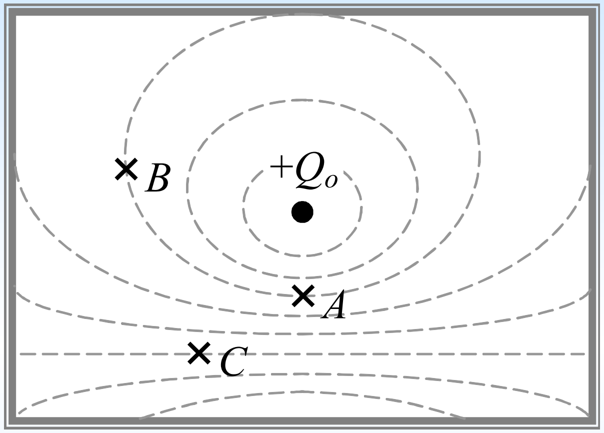
Do you *agree* or *disagree* with this statement?

- If you agree, state so explicitly. Explain your reasoning.
- If you disagree, use your results from this tutorial to provide <u>**at least three (3)**</u> specific counterexamples. Explain your reasoning.

Examples of assessment questions

On written exams after modified instruction

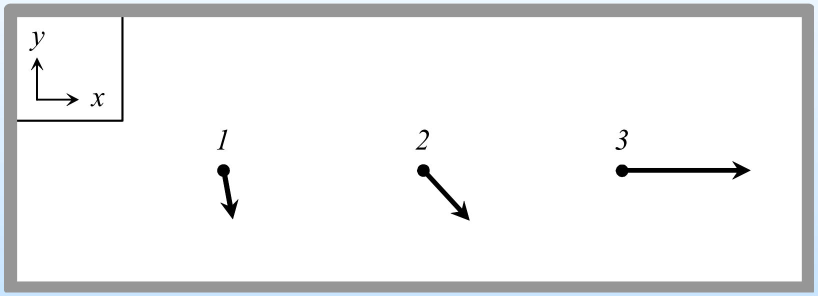
Task: Given equipotential map, predict directions and relative magnitudes of forces.



GVSU: **20/23 correct** (2 classes)

SPU: **8/11 correct** (1 class)

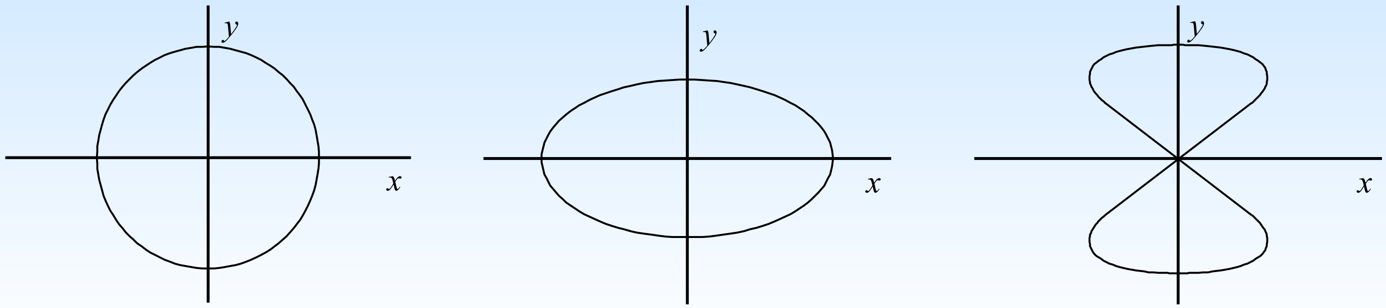
Task: Given several forces, sketch a possible equipotential map and rank points by potential energy.



GVSU: **14/30 correct** (3 classes)

Tutorial: ***Harmonic motion in two dimensions***

Connecting force constants and frequencies



Case #1 **Case #2** **Case #3**

Students are guided to recognize:

- how many oscillations occur along the *y*-axis for each oscillation along the *x*-axis
- how differences in force constants affect periods and frequencies
- how phase difference between *x*- and *y*-motions affect trajectories of isotropic oscillators

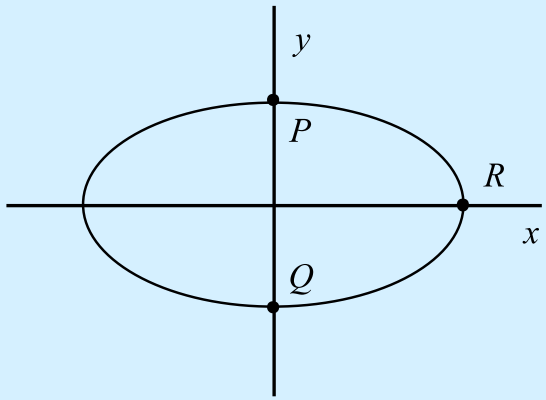
Tutorial homework: ***Harmonic motion in two dimensions***

Connecting amplitude and potential energy (<u>not</u> frequency)

Excerpt from tutorial homework:

A. Critique the following statement. Explain.

> "The oscillator goes farther in the *x*-direction than in the *y*-direction. That means the spring in the *y*-direction must be stiffer than the spring in the *x*-direction."

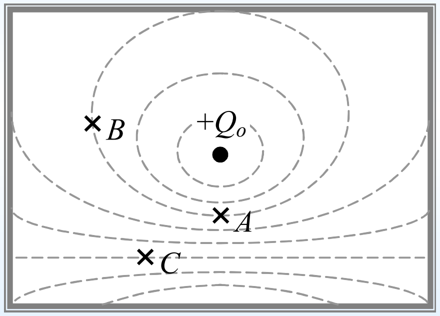


B. Rank points *P, Q,* and *R* according to (i) total energy, (ii) potential energy, (iii) kinetic energy.

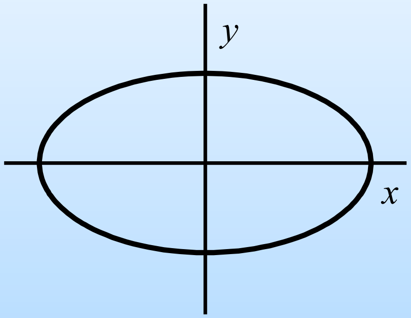
Explain how the difference in the *x*- and *y*-amplitudes, used *incorrectly* in the statement in part A, can help justify a *correct* answer here in part B.

Summary and reflections

- Physics majors in *advanced* courses can and do experience conceptual and reasoning difficulties similar to those already identified at the *introductory* level.
  - Difficulty discriminating between a **quantity** and its **rate of change**



  - Reliance on inappropriate **“compensation arguments”** with two or more variables



Summary and reflections

- Specific difficulties must be addressed *explicitly* and *repeatedly* for meaningful learning to occur.

- Students need guidance to extract physical meaning from the mathematics.
  - **Guided sense-making** seems more important than derivations.
  - Students need practice articulating **in their own words** the physical meaning expressed in the *graphical representations* and in the *mathematics* they use.

Intermediate Mechanics Tutorials

Project website: **http://perlnet.umaine.edu/IMT**

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Supported by NSF grants DUE-0441426 and DUE-0442388

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