1. Use your knowledge of the properties of ellipses (see p. 1 of the tutorial to remind yourself of these) in order to show that the semi-minor axis can be written in terms of the semi-major axis a and eccentricity ε as follows:

$$b = a(1 - \varepsilon^2)^{1/2}$$

2. For this problem we need to "tie up" some loose ends from the tutorial. In particular, we need to show that the radial equation of motion (using Newton's second law):

$$m\left(\ddot{r} - r\dot{\theta}^2\right) = -\frac{GMm}{r^2} \qquad \text{(Eq. 3 from tutorial)}$$

is equivalent to the following differential equation using a change of variables:

$$u(\theta) \equiv \frac{1}{r(\theta)} \quad \Rightarrow \quad \frac{d^2u}{d\theta^2} + u = \frac{GM}{l^2} \qquad \text{(Eq. 4 from tutorial)}$$

a. First, show that the term $r\dot{\theta}^2$ from the first equation can be rewritten as u^3l^2. (*Hint:* Apply your results from section II.A of the tutorial.)

b. Next, we need to replace $\ddot{r}$ from the first equation with an expression in terms of l, $u(\theta)$, and derivatives of u.

i. Let's take the first time derivative of $r = r(\theta)$. (Keep in mind that we are treating r as a function of θ, but θ is *implicitly* a function of time too!)

Carefully verify and justify each step in the following: $\dot{r} = \frac{d}{dt}\left(\frac{1}{u}\right) = -\frac{1}{u^2}\frac{du}{d\theta}\dot{\theta} = -l\frac{du}{d\theta}$.

ii. Take one more time derivative, to turn $\dot{r}$ into $\ddot{r}$, and in so doing show how to express $\ddot{r}$ in terms of $d^2u/d\theta^2$, u, and l, but **<u>not</u>** $\dot{\theta}$.

c. Now combine your results from parts b.i and b.ii to show that the radial equation of motion (Eq. 3 from tutorial) is equivalent to the simpler looking differential equation of motion shown in Eq. 4 from tutorial.