INSTRUCTOR NOTES
Energy and angular momentum for elliptical orbits

Emphasis

In this tutorial students apply energy and angular momentum concepts to solve qualitative and quantitative problems in orbital mechanics. The tutorial provides students the opportunity to interpret and apply two fundamental relationships that link constants of the motion to physical parameters of elliptical orbits:

Angular momentum per unit mass l and latus rectum α: $$l = \sqrt{GM\alpha} \qquad (1)$$

Total energy E_{tot} of two-body system and semi-major axis a: $$E_{tot} = -\frac{GMm}{2a} \qquad (2)$$

Prerequisites

Students will need to have completed instruction on the solutions to the polar equation of an orbit (which yields Eq. 1 above) and to the energy equation of an orbit (which yields Eq. 2). It is also highly recommended that students have already completed the following: the tutorial <u>and</u> tutorial homework for *Angular momentum and Kepler's second law,* the tutorial <u>and</u> tutorial homework for *Gravitation and conservation of energy.*

TUTORIAL PRETEST

The pretest consists of several tasks each involving the same set of three possible orbits that a shuttlecraft follows, one at a time. Two orbits are circular with different radii, and the third is elliptical with the same semi-major axis as one of the circular orbits (although this fact is not pointed out explicitly). The students are asked to rank the three orbits according to the following quantities and to explain their reasoning: (a) eccentricity, (b) total energy of the planet-shuttle system, and (c) angular momentum of the shuttle (measured with respect to the center of the planet). These tasks probe the ability of students to understand and apply the relationships stated in Equations 1 and 2.

TUTORIAL SESSION

Equipment and handouts

Each group will need a whiteboard and set of markers, or a large sheet of paper. Each student will need a copy of the tutorial handout (no special handouts required).

Discussion of tutorial worksheet

Section I: Energy, angular momentum, and the shapes of elliptical orbits

The two brief "warm-up" questions in this opening section direct the attention of the students to the one-to-one connections between l and α and between E_{tot} and a expressed in Equations 1 and 2 above. It may be advisable to do an informal "checkpoint" with each group immediately after completing this section. (If the class is large, or if most groups finish this section at the same time, a brief discussion with the entire class may suffice.)

Section II: Application: Transfer from an elliptical to a circular orbit

In this section students consider a shuttlecraft orbiting a planet whose orbit is initially elliptical in shape. They consider qualitatively how to maneuver the shuttle into a circular orbit from the apogee of the original orbit. They justify their answer using two independent arguments: one involving energy and one with angular momentum. Students then answer the same questions assuming that it is desired to maneuver the shuttle into a circular orbit from perigee instead of from apogee.

Even though students have already completed section I by this point, they may not recognize how to apply the results from that section. For students who do not how to proceed, ask them to sketch the desired orbit to which they are imagining the shuttlecraft to transfer. Then ask the students to notice any differences in the shapes of the orbits. Students should be able to recognize that any change to the latus rectum or semi-major axis is relevant and proceed on the basis of the appropriate relationship from Equation 1 or 2. If possible, wait until the students themselves refer to the latus rectum or semi-major axis before mentioning them yourself.

Section III: Application: Comparison of circular and elliptical orbits

Students are now a posed a series of qualitative problems loosely based on the pretest. They are shown an orbit for a shuttlecraft around a planet that is similar to the single elliptical orbit from the pretest task. In part A students must identify the circular orbit corresponding to the same total energy as the original elliptical one. Many students recognize immediately that the semi-major axes should be equal, but watch for students who either miscount the number of distance units for the semi-major axis or who draw the circular orbit as being centered on the geometric "center" of the elliptical orbit rather than centered on the planet. After identifying the correct intersection points of the orbits, students are asked which orbit (if any) would result in the shuttle having the faster speed at an intersection point. Finally, students identify which orbit corresponds to the greater angular momentum and to justify their answer with two independent arguments—first using the definition of angular momentum and then comparing the latus rectum values of the orbits.

Part B asks students to identify another circular orbit, this time one that corresponds to the same angular momentum of the shuttlecraft as the original elliptical one. As before, students determine which orbit would result in the shuttle having the faster speed at an intersection point, and they justify their answer with two independent arguments—one using the definition of angular momentum and another using energy ideas.

It is possible that bright students may finish the tutorial with time remaining. Encourage these students to begin the homework, which contains several problems that build upon the results obtained in section II and III (see below).

TUTORIAL HOMEWORK

The homework includes additional applications, both qualitative and quantitative, that require students to extend their results from tutorial.

1. The first problem requires students to use basic features of ellipses to express the apogee and perigee distances in terms of the eccentricity and either the semi-major axis or latus rectum. Although this problem is indirectly related with the emphasis of the tutorial, it sets the stage for the derivations and calculations in the remaining problems.
2. Students revisit the elliptical orbit from section II of the tutorial. Starting with Eq. (1) they derive algebraic expressions for and calculate the speed of the shuttle at apogee and perigee. They also calculate other parameters that describe the orbit.
3. In this multi-part problem students examine the physics of Hohmann transfer orbits. In part a students must explain qualitatively the required maneuvers to enter and leave the transfer orbit. Part b asks for calculations of the transfer orbit, including the time required to traverse the transfer orbit. Part c motivates the idea of "launch opportunities" for expeditions that require the use of Hohmann transfer orbits. Finally, part d requires students to calculate the changes in speed at the entry and exit points of the transfer orbit.
4. The final problem gives students the opportunity to expand upon their results from section III of the tutorial, in which they identified pairs of orbits corresponding to the same total energy

or the same angular momentum. On the basis of the scale diagrams they drew during tutorial, students calculate the angle formed by each pair of orbits at a point where they intersect.