ENERGY AND ANGULAR MOMENTUM FOR CLOSED ORBITS

I. Energy, angular momentum, and the shapes of elliptical orbits

Imagine that you were to pilot a shuttlecraft in an elliptical orbit around an uncharted planet. Your orbit has semi-major axis a_1, semi-minor axis b_1, and latus rectum α_1. Another (identical) shuttle enters a different elliptical orbit with corresponding quantities a_2, b_2, and α_2.

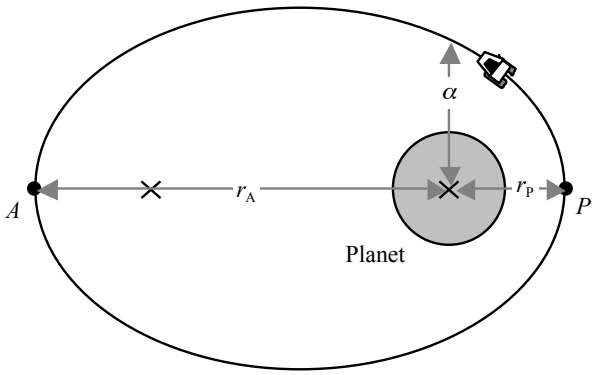
A. If the orbits of both shuttles correspond to the same total energy, which of the quantities mentioned above must be *equal?* Explain your reasoning.

B. How would your answer to part A be different if the orbits instead corresponded to the same angular momentum? Explain.

II. Application: Transfer from an elliptical to a circular orbit

Suppose that the orbit of your shuttlecraft were shown in polar view below. The foci of the orbit (each indicated by an "×"), perigee distance (r_P), apogee distance (r_A), and latus rectum *(α)* are labeled.

A. It is desired to transfer from the original elliptical orbit to a circular one. To conserve fuel, you want to accomplish the maneuver with a *single* firing (either forward or reverse) of your shuttle's thrusters.



Polar view of orbit *(not to scale)*

To maintain a constant distance r_A from the center of the planet (starting at point *A*), would you need to *increase* or *decrease* the speed of the shuttle as it passes point *A?* Justify your answer two different ways:

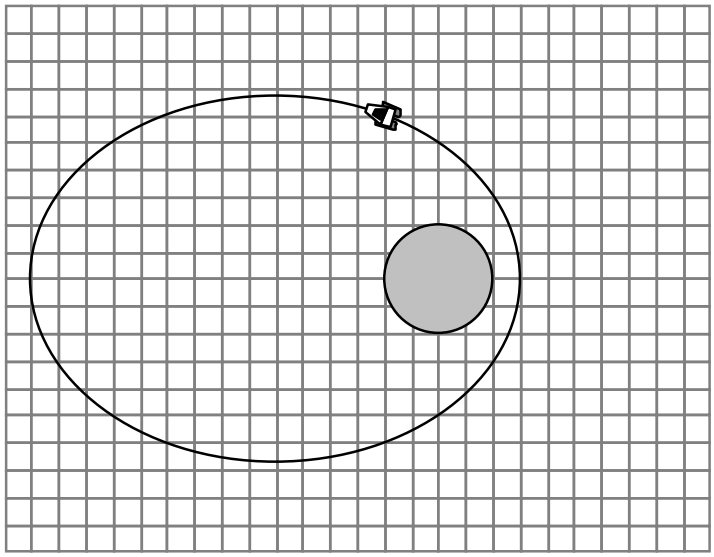
- using an argument based on angular momentum

- using an argument based on the total energy of the shuttle-planet system

B. How (if at all) would your answer in part A be different if you instead wanted to achieve a circular orbit of radius r_P starting from point *P?* Explain.

III. Application: Comparison of circular and elliptical orbits

Suppose now that the orbit of your shuttle is shown in polar view at right.



A. Carefully and accurately sketch the *circular orbit* in which the shuttle-planet system would have the same total energy as for the original orbit. Explain.

Consider a location where both orbits intersect.

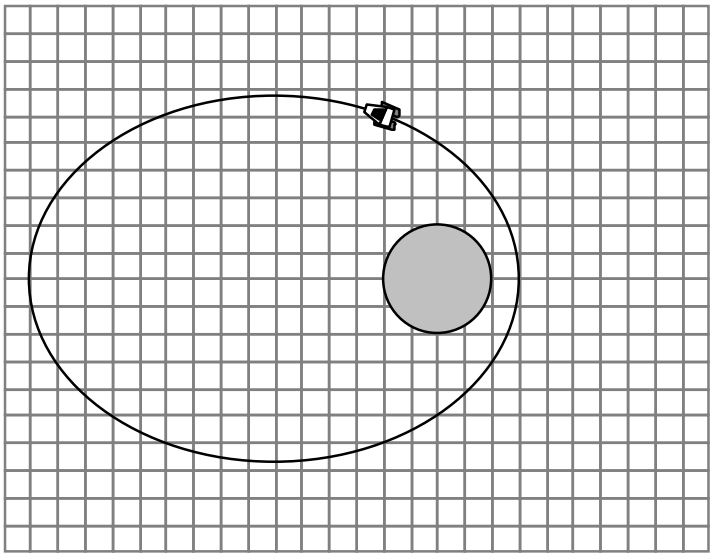
1. In which orbit (if any) would the shuttle pass through that location with the faster speed? Explain how you can tell.

2. In which orbit (if any) would the shuttle pass through that location with the greater angular momentum? Justify your answer two different ways:

   - using your answer in part 1 as a guide, and

   - using the shapes of the orbits as a guide

B. Carefully and accurately sketch the circular orbit in which the shuttle would instead have the same angular momentum as it did in the original orbit. Explain your reasoning.



Consider a location in space where both orbits intersect. In which orbit (if any) would the shuttle pass through that location with the faster speed? Justify your answer two different ways:

- using your knowledge about the angular momentum for each orbit, and

- using your knowledge about the total energy (of the shuttle-planet system) for each orbit

✓ **STOP HERE** and check your results with an instructor.

