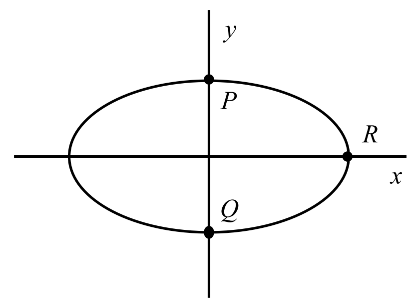
1. The potential energy function for a 2-D oscillator may be written: $U(x, y) = \frac{1}{2} k_1 x^2 + \frac{1}{2} k_2 y^2$.

   Show that component differential equations of motion are ***separable,*** and explain why the position of the oscillator can be written as:

$$x(t) = A_1 \cos(\omega_o t + \varphi); \quad y(t) = A_2 \cos(\omega_o t + \varphi + \delta)$$

2. Shown at right is the *x-y* trajectory for a 2-D oscillator.



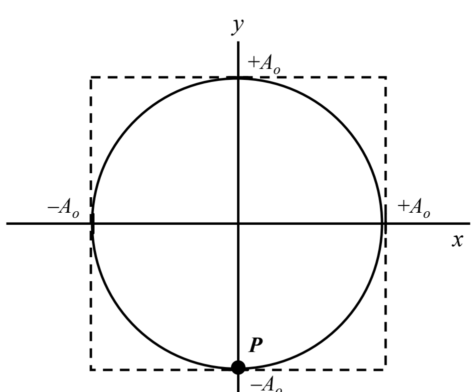
   a. Consider the following *incorrect* statement:

   "The oscillator goes farther in the *x*-direction than in the *y*-direction. That means the spring in the *y*-direction must be stiffer than the spring in the x-direction."

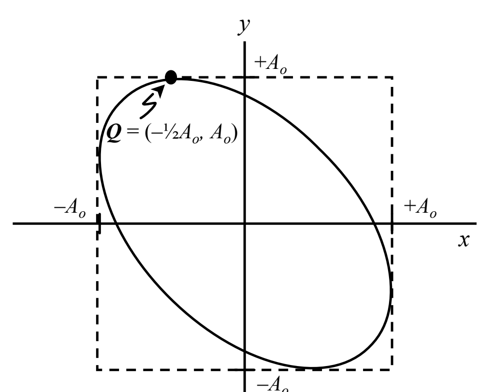
   Identify the error in the above statement, and state how you would modify it to make it correct.

   b. Rank the labeled points *(P, Q,* and *R)* according to: (i) total energy, (ii) potential energy, (iii) kinetic energy. Explain how the difference in amplitudes in the *x*- and *y*-directions, used *incorrectly* in the statement in part a, can be used to justify a *correct* conclusion in this part of the problem.

3. Each diagram below describes the motion of a 2-D harmonic oscillator.



Case #1: Oscillator starts at *P* with initial speed $5A_o$ m/s to the right



Case #2: Oscillator starts at *Q,* goes clockwise, and returns to *Q* 4π sec later

For each case, determine the exact expressions *x(t)* and *y(t)* that describe the position of each oscillator as functions of time. (Use the functional forms for *x(t)* and *y(t)* shown in Problem 1 of this homework.) Explain your reasoning and show all work.

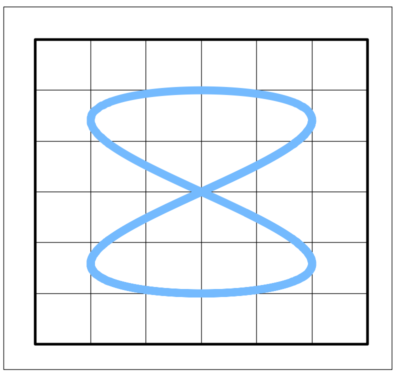
4. Consider the motion of a two-dimensional oscillator, $U(x, y) = ½\, k_1x^2 + ½\, k_2y^2$, in which the spring constants are *unequal:* $k_1 \neq k_2$.

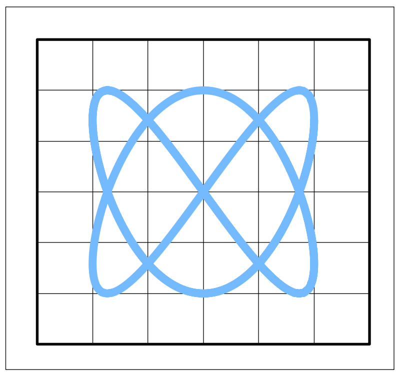
   a. Justify the term *non-isotropic* as it applies to such an oscillator.

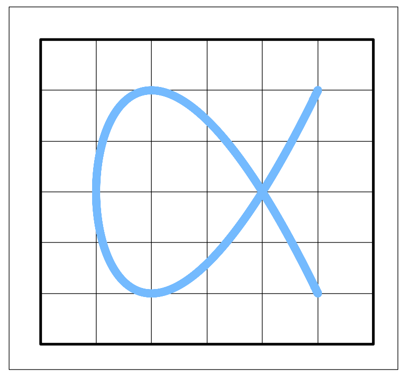
Each trajectory below depicts the possible motion of a unique oscillator. All three oscillators, however, share the property that the angular frequencies ω_1 and ω_2 for the motions along the *x*- and *y*-axes are *commensurate, i.e.,* that the angular frequencies satisfy the following relationship:

$$\frac{\omega_1}{n_1} = \frac{\omega_2}{n_2}, \text{ where } n_1 \text{ and } n_2 \text{ are integers.}$$

   b. For each case below, (i) determine whether ω_1 is *greater than, less than,* or *equal to* ω_2, and (ii) determine the values n_1 and n_2 that satisfy the condition shown above. Explain.


Trajectory #1


Trajectory #2


Trajectory #3

   c. In **Trajectory #1** above, suppose that the oscillator has a mass of 0.20 kg and retraces its path every 4π seconds. Determine the numerical values of ω_1, ω_2, k_1 and k_2 for that case. Show all work.

