Emphasis

Students examine qualitatively and quantitatively the motion of a simple harmonic oscillator, as exemplified by a mass attached to an ideal spring. Students use a strobe picture of the oscillator's motion to review and apply the vector kinematics definitions of velocity and acceleration. They show that their results are consistent with Newton's Second Law. Students then construct and analyze the differential equation of motion in order to isolate those factors (mass and force constant) that affect the frequency. They also discuss possible solutions to the differential equation for specific cases.

Prerequisites

This tutorial comes first among the conceptual development tutorials in *Intermediate Mechanics Tutorials,* and thus no other tutorials from this collection are required beforehand. However, students should be familiar with Hooke's law and basic terms used to describe simple harmonic motion (*e.g.,* amplitude, period, frequency).

TUTORIAL PRETEST

The first pretest problem consists of a basic question that asks students to determine the period of one instance of simple harmonic motion given a strobe "photograph" of the motion and the time interval between consecutive pictures. This problem serves as a baseline for determining whether or not students are familiar with the term *period.*

The second problem consists of a set of questions. Each question describes one possible change to the physical situation described in the first problem. For each change, the students are asked to determine whether that change would cause the period to be longer than, shorter than, or equal to what it was in the original case. Students consider the effect of varying: (1) the initial displacement of the block attached to the spring (measured from equilibrium), (2) the stiffness of the spring, and (3) the mass of the block. Although students may be able to use their experience from introductory physics to give correct responses, few students answer all parts correctly. The most difficult part is that which asks about changing the initial displacement of the block, and hence the amplitude of the oscillation; many incorrectly predict that increasing the amplitude will cause an increase in period (*e.g.,* "the block must travel farther in each cycle").

The final problem is a task that probes the ability of students to interpret the *x vs. t* graphs of two oscillators that are out of phase with each other. The students are asked specifically to determine the amount by which the oscillators are out of phase and to specify which oscillator is ahead of the other.

TUTORIAL SESSION

Equipment and handouts

Each group will need a whiteboard and set of markers, or a large sheet of paper. Each student will need a copy of the tutorial handout (no special handouts required).

Optional demonstration equipment: Stopwatch, support rod and stand, one or more springs, variety of hanging masses (including two masses that differ by a factor of 4). This equipment could be used for a demonstration after all groups have reached the checkpoint on page 2.

Discussion of tutorial worksheet

Section I: Qualitative analysis of motion

This first section of the tutorial is specially designed to help students review and apply the vector kinematics definitions of (instantaneous) velocity and acceleration. Starting with a strobe picture of the motion of a block attached to an ideal spring, students draw vectors for the velocity of the block at each instant shown, use vector subtraction to determine the direction of average acceleration between instants, and refine their method (considering infinitesimal time intervals) so that it yields the direction of the instantaneous acceleration at each instant. Use of such vector constructions should address lingering difficulties students may have with velocity and acceleration, *e.g.,* the incorrect belief that the acceleration must be zero if velocity is also zero. Students verify that their results are consistent with the net force on the mass at each instant.

Section II: Differential equation of motion

In parts A and B of this section, students consider the same motion illustrated in the strobe diagram from section I. They use Newton's Second Law to write down the differential equation of motion for the situation. The students are then prompted to show using direct substitution that $x(t) = A\cos(\omega t + \phi_o)$ and $x(t) = A\sin(\omega t + \phi_o)$ are possible solutions to the differential equation, as long as the angular frequency ω satisfies the expected condition $\omega = (k/m)^{1/2}$.

Even though students may have had prior instruction on differential equations, many might not recognize what is required in proving "by direct substitution" that a given function solves a differential equation. Some students may even find strange the idea that a "solution" in the context of differential equations is a *function* rather than a numerical value; these students may take "equation of motion" to mean the solution $x(t)$ rather than the differential equation for which it is a solution. Student groups who have difficulty recognizing how to proceed in part B may be experiencing basic difficulties such as these.

After completing part B, students should recognize that only the spring constant and the mass of the block, not the amplitude of the motion, affect the frequency (and thus the period). The questions in part C require students to utilize this result. Students predict how the period is affected by each of three possible changes (to amplitude, spring constant, or mass). Make sure that students heed the checkpoint at the end of section I. Instructors should carefully check that students explain effectively how their results from part B helped them decide their answers in part C. The most likely place for errors is the case in which the oscillator mass quadruples; some may not realize that the period must not only decrease but do so by a factor of two.

Section III: Expressing position as a function of time

In this section students use multiple forms of the solution $x(t)$ to describe completely, including initial conditions, the motion of a simple harmonic oscillator. Taking the motion shown in the strobe diagram from section I, the students must calculate the values of A, ω, and ϕ_o appropriate for both the cosine and sine forms of the solution *(i.e.,* both $x(t) = A\cos(\omega t + \phi_o)$ and $x(t) = A\sin(\omega t + \phi_o)$).

Students should have little difficulty determining the amplitude (0.5 m) or period (0.80 s) of the motion. However, watch for errors that might arise from an underlying confusion between period and frequency. Many students may hold the notion that frequency f is "defined" as "$f = 1/T$" rather than as the number of cycles occurring in a given unit time. Some students may treat the period as identical in meaning to either f or ω. Others may not recognize how frequency f is related to angular frequency $(\omega = 2\pi f)$.

Regarding the initial phase angle ϕ_o, watch for students who say that the sine version of the solution requires an initial phase angle of $-\pi/2$ (rather than the correct value of $+\pi/2$). These students often explain that a pure sine curve must be shifted "in the negative direction" by an amount $\pi/2$ in order to match a pure cosine curve. (The last pretest task is designed to probe for this and similar types of incorrect reasoning.) The "student dialogue" in part B is designed to help students recognize and remedy this error should it arise. The checkpoint at the end of section II should be enforced to ensure that students can articulate a clear and convincing argument that goes beyond a memorized mathematical algorithm.

TUTORIAL HOMEWORK

The homework gives students the opportunity to apply and extend their results from the tutorial. Some problems place particular emphasis on constructing appropriate differential equations of motion and recognizing the form of differential equation that describes simple harmonic motion.

1. Students gain practice in expressing $x(t)$ and $v(t)$ for another example of simple harmonic motion, similar to that given in section II of the tutorial.
2. Students analyze a simple pendulum undergoing small oscillations in one dimension.
3. Students apply vector superposition of forces and Newton's law of gravitation to analyze another situation that gives rise to oscillatory motion. The students are expected to recognize that small oscillations may be approximated as simple harmonic oscillations, and further, the students must quantify "how small is small" to make this approximation.
4. Students revisit the last pretest problem, which tests the ability to determine from the motion graphs of two oscillators the phase relationship between them.
5. This last homework problem is an extension problem in which students determine the effective force constant for springs "in series" and "in parallel."

