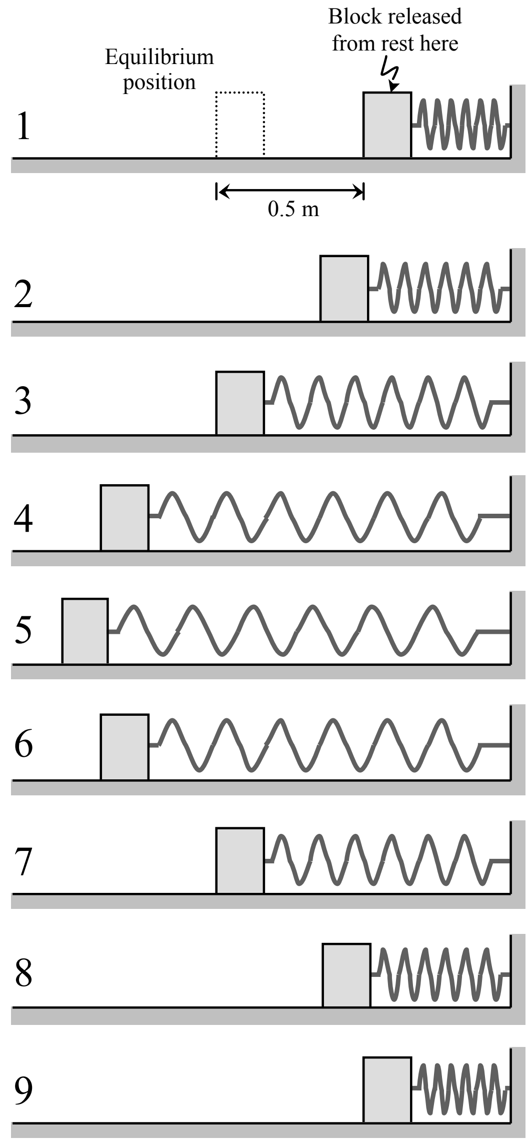
SIMPLE HARMONIC MOTION

I. Differential equation of motion



A block is connected to a spring, one end of which is attached to a wall. (Neglect the mass of the spring, and assume the surface is frictionless.)

The block is moved 0.5 m to the right of equilibrium and released from rest at instant 1. The strobe diagram at right shows the subsequent motion of the block (*i.e.,* the block is shown at equal time intervals).

A. Using Newton's second law in one dimension, $F_{net} = m\ddot{x}$, write down the differential equation that governs the motion of the block.

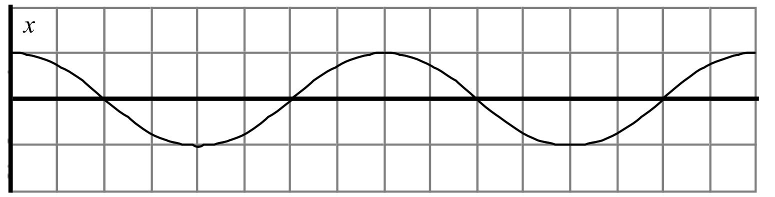
The net force exerted on the block may be called a *restoring force.* Justify this term on the basis of your differential equation above.

B. Show by direct substitution that the functions $x(t)$ given below are solutions to the differential equation you wrote down in part A.

As part of your answer, specify the conditions (if any) that must be met by the parameters A, ω, and ϕ_o in order for each function to be a valid solution.

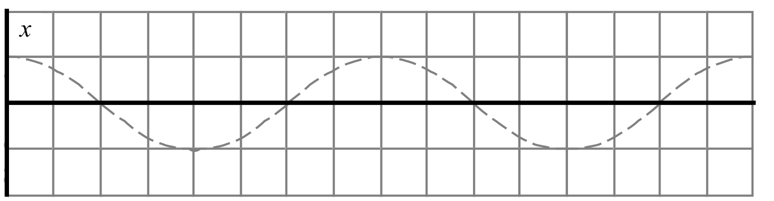
- $x(t) = A\cos(\omega t + \phi_o)$
- $x(t) = A\sin(\omega t + \phi_o)$

C. Shown at right is the x vs. t graph representing the motion of the block described on the preceding page. Note that $t = 0$ corresponds to the instant ("instant 1") when the block is released from rest.

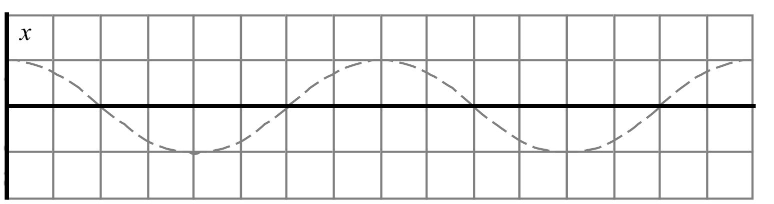


Suppose that the experiment described in section I were repeated exactly as before, **except** with one change to the setup. For each change described below, sketch the new x vs. t graph for the block. Show as much detail as possible in your new graph. Use your results from part B (on the preceding page) to justify your answers.

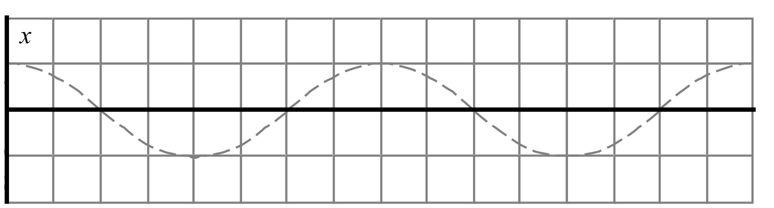
1. The spring is replaced with a stiffer spring.



2. The block is replaced with another block with four times the mass as the original one.



3. The block is released 0.25 m to the left of equilibrium (instead of 0.5 m to the right of equilibrium).



✓ **STOP HERE** and check your results with an instructor.

II. Expressing position as a function of time

A. Consider again the motion of the block in section I, including the initial conditions of the motion. Suppose that, in the strobe diagram shown in section I, an interval of 0.10 s elapsed between consecutive pictures.

For each of the functions you examined in part B of section I (listed below), evaluate all constant parameters (A, ω, and ϕ_o) so as to completely describe the position of the block as a function of time.

- $x(t) = A \cos(\omega t + \phi_o)$

- $x(t) = A \sin(\omega t + \phi_o)$

B. Check your answers in part A above by examining the dialogue below.

Chris: "The cosine function is the same as a sine curve that has been shifted along the time axis to the left by $\pi/2$ radians."

Pat: "That's right. That means that the function cos ωt is identical to $\sin(\omega t - \pi/2)$, because the phase shift of $-\pi/2$ shifts the sine curve to the left by $\pi/2$."

Chris' statement is *correct,* however Pat's response is *incorrect.* Identify the error in Pat's reasoning and describe how you would modify Pat's statement so that it would be correct.

C. Are your answers in parts A and B consistent with each other? If not, resolve the inconsistencies.