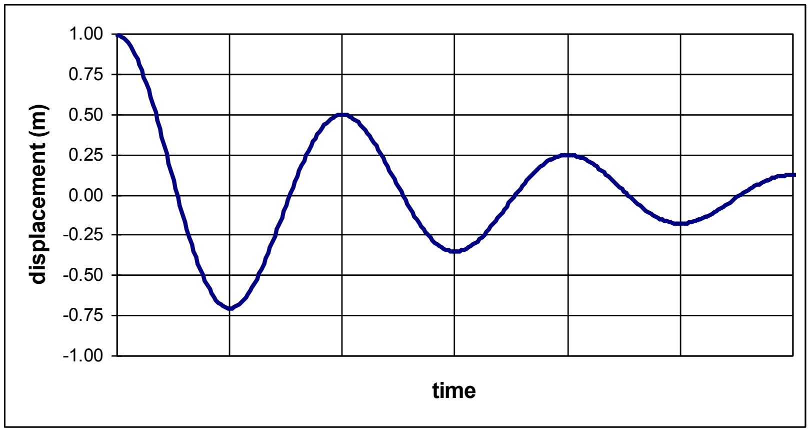
DAMPED HARMONIC MOTION: MOTION GRAPHS

I. Displacement versus time

Consider a simple harmonic oscillator (*e.g.,* a mass connected to an ideal spring) that experiences a retarding force that is always proportional to the speed of the oscillator.

At $t = 0$ the oscillator is displaced 1.0 m away from equilibrium and released from rest. The displacement versus time *(x vs. t)* graph at right represents the subsequent motion of the oscillator.

(Because the motion still exhibits oscillatory behavior, the oscillator is said to be *underdamped*).



A. According to the graph, how (if at all) does each of the following quantities change as time elapses?

- the maximum displacement attained with each oscillation

- the period of oscillation

B. Suppose that the retarding force were removed (*e.g.,* the oscillator is now immersed in a vacuum rather than air). Imagine that the oscillator is now released with the *same initial conditions* as before.

   How, if at all, would removing the retarding force affect each of the following quantities? Discuss your reasoning with your partners.

   - the net force exerted on the oscillator when it is located somewhere between $x = +1.0$ m and $x = 0$

     (*Hint:* Drawing free-body diagrams will help!)

   - the acceleration of the oscillator when it is located somewhere between $x = +1.0$ m and $x = 0$

   - the amount of time required for the oscillator to travel from $x = +1.0$ m to $x = 0$

C. Your answers in part B above suggest that retarding force will change the period of oscillation. Use your results to predict whether the period of the ideal (frictionless) oscillator is *longer than* or *shorter than* that of the damped oscillator. Explain.

On the graph on the preceding page, illustrate your prediction by sketching a qualitatively correct x vs. t graph for the ideal (frictionless) harmonic oscillator. Assume that the initial conditions of the motion are the same as before (*i.e.,* the oscillator is released from rest at $x = 1.0$ m at $t = 0$).

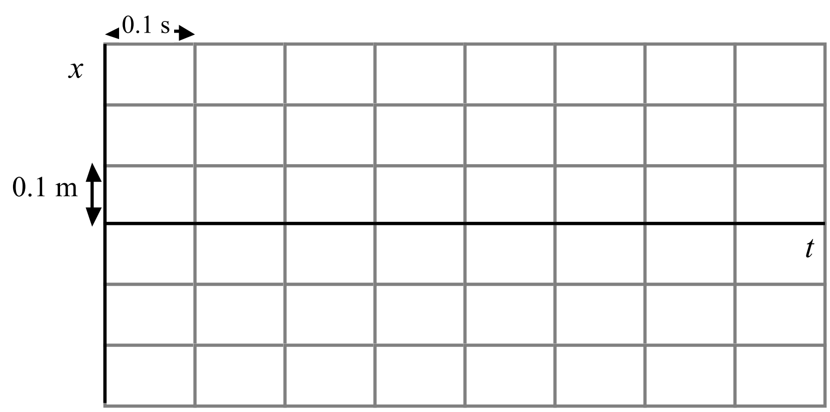
D. Summarize your results thus far: If an ideal (frictionless) harmonic oscillator is subjected to a retarding force proportional to its speed, how is the frequency of the oscillator affected? Explain.

✓ **STOP HERE** and check your reasoning with an instructor.

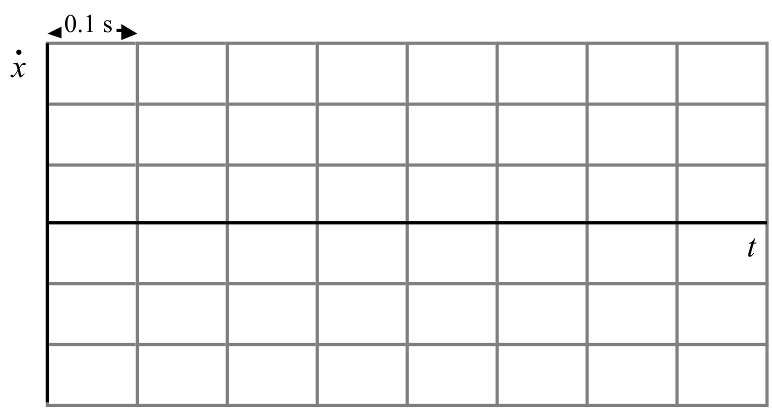
II. Velocity versus time

Consider now another underdamped harmonic oscillator with a period of 0.4 s. At $t = 0$ the oscillator is released from rest at $x = 0.2$ m.

A. In the space at right, sketch a position *vs.* time graph for the oscillator. (Note that each grid corresponds to 0.1 m of distance and 0.1 s of time.)



B. On the next set of axes, carefully sketch a qualitatively correct graph of velocity *vs.* time for the oscillator.



C. Consider the instant (call it t_o) at which the oscillator first passes through $x = 0$. Identify this instant on both graphs on the preceding page.

1. Is the net force exerted on the oscillator equal to *zero* when it passes through $x = 0$? If not, is the net force in the *same direction* or in the *opposite direction* as the velocity? Explain.

   (*Hint:* Draw a free-body diagram for the oscillator for the instant when it passes through $x = 0$.)

2. On the basis of your answer in part 1:

   - At $t = t_o$, is the oscillator moving with *increasing speed, decreasing speed,* or *neither?* Explain.

   - Does the oscillator first reach a maximum speed *at* $t = t_o$, *after* $t = t_o$, or *before* $t = t_o$? Explain.

D. Is your velocity vs. time graph from part B (on the preceding page) consistent with your results in part C above? If not, resolve the inconsistencies.

✓ **STOP HERE** and check your results with an instructor.

