INSTRUCTOR NOTES

Conservative forces and equipotential diagrams *(topo map version)*

Emphasis

This tutorial is designed to address specific conceptual and reasoning difficulties in interpreting equipotential diagrams and understanding how a conservative force field is related to the corresponding potential energy function.

Prerequisites

It is recommended that students have covered vector curl, gradient, and the properties of conservative forces *(i.e.,* work is path-independent, $\vec{\nabla}\times\vec{F}=0$, $\vec{F}=-\vec{\nabla}U$ *)* before this tutorial.

TUTORIAL PRETEST

The pretest probes the ability of students to use equipotential diagrams to infer information about the corresponding (conservative) force. Specifically, the students are shown an equipotential map with three labeled points and are asked (1) to draw an arrow at each point to indicate the direction in which a test particle would move when released from rest, and then (2) to rank the labeled points according to the magnitude of the force exerted on the test particle at those points. The task is posed in an electrostatic context (*e.g.,* the test particle is described as a test charge) so that it resembles similar tasks appropriate for an introductory electromagnetism course. In so doing, the task is made both accessible to students and effective in probing their qualitative understanding of force as opposite the gradient of potential energy.

Students are usually more likely to indicate the correct direction of the force at each point than they are to rank the relative magnitudes of the forces. Expect no more than 1/3 of the students to give a correct ranking, with many (if not most) incorrect rankings based on the incorrect belief that the magnitude of the force at a given location is proportional to the value of potential energy at that location. Regarding the directions of the forces, some may sketch force vectors parallel to, rather than perpendicular to, the local equipotentials. Others may infer from the equipotential contours that a single point charge is located just off the left edge of the diagram and draw their force vectors based on the notion that that charge is the only source charge present, even though the contours are not concentric circles.

TUTORIAL SESSION

Equipment and handouts

Each group will need a whiteboard and set of markers, or a large sheet of paper. Each student will need a copy of the tutorial handout (no special handouts required).

Discussion of tutorial worksheet

Section I: Topographical maps and equipotential contours

The tutorial opens by asking students to interpret a portion of a topographic map with several locations labeled. When working in groups, students demonstrate little difficulty recognizing that the proximity of elevation lines at a given location determines the slope—and hence the magnitude of the net force—at that location. They also should recognize that direction to be taken as “directly downhill” from each location—and hence the direction of the net force—is oriented perpendicular to the local elevation lines. Finally, students consider the motion of identical boulders moving (without friction) from rest down different slopes but experiencing the same drop in elevation. Students should be able to answer that the steeper slope will cause the greater acceleration but, due to either conservation of energy or the work-energy theorem, each will gain the same amount of kinetic energy. The checkpoint at the end of this section should be used to verify that students understand these basic elements of topographic maps, upon which the following section will build.

Section II: Forces and changes in potential energy

In part A of this section students make an explicit analogy between a topographical map and a (two-dimensional) equipotential diagram by assuming the flat-earth approximation of gravitational potential energy, $U^G(x, y) = mgh(x, y)$. Students then continue in parts B and C by being guided to construct an operational definition of the gradient of the potential energy. In part D students interpret their results regarding both the magnitude and direction of the gradient. Finally, in part E students recognize that the direction of the force always points opposite the gradient of potential energy and that the magnitude of the force should vary with the gradient of potential energy (rather than proportional to just potential energy).

Part A usually presents few obstacles for students, but in part B students sometimes give incomplete verbal descriptions of how to use a topographical map to calculate the partial derivatives ($\partial U/\partial x$ and $\partial U/\partial y$) that will comprise the components of the gradient vector. An acceptable description for $\partial U/\partial x$, for example, would be as follows: "Given the desired (x_o, y_o) consider another location $(x_o + \partial x, y_o)$ located a distance ∂x away (or, another location ∂x to the 'east' of the original location). The partial derivative $\partial U/\partial x$ would be the difference in potential energies at these two locations, taking 'second point minus the first,' and then dividing this difference by ∂x." Watch for students who neglect to specify the order of subtraction or who neglect the step of dividing the difference in potential energy values by the distance ∂x.

Students then proceed to part C by applying their results in part B to the same three locations labeled on the topographical map used in section I. Even though there is no "official" checkpoint, it is wise to check students' results in part C, especially those tabulated on page 3 of the tutorial, and to ask students to explain their reasoning. They should find that their vectors for the gradient of potential energy point directly uphill from each labeled location—*e.g.,* at location *C* the gradient has a negative-*x* component and an even larger component in the positive-*y* direction.

Parts D and E guide students to recognize how their results earlier in section II relate to their findings from section I. In part D students are asked explicitly whether locations of equal potential energy must also have the same magnitude $\vec{\nabla} U$; make sure students can adequately explain their reasoning here, so that they will then recognize that force is not proportional to potential energy. Although students do not *prove* the relationship $\vec{F} = -\vec{\nabla} U$ here, they are asked to demonstrate how their results from the tutorial are consistent with this relationship.

(*Note:* For bright students who finish the tutorial with time remaining in class, you may wish to ask a "challenge question" such as this: "In this tutorial we have made an analogy between topographic maps and equipotential diagrams to help us understand what the equation $\vec{F} = -\vec{\nabla} U$ means. Can you think of a way in which this analogy might break down?" It may help to nudge students to think of extreme cases, such as would be the case a topographic map of a vertical cliff: the force remains finite but $\vec{\nabla} U$ becomes infinite in magnitude.)

TUTORIAL HOMEWORK

The homework gives students the opportunity to apply and extend their results from the tutorial. It also includes problems that attempt to address the particularly tenacious incorrect belief that force is always proportional to potential energy.

1. Students revisit the equipotential map problem from the in-class pretest. This problem therefore serves as a direct application of the ideas developed in the tutorial.
2. Students examine four vector force fields and determine whether or not a self-consistent set of equipotential contours can be drawn for each case. Thus this problem requires students to

stretch their thinking from tutorial—in which they started with an equipotential (topographical) map and inferred information about the force—and "work backwards."

3. This problem forces students to critique an (incorrect) statement about the relationship between force and potential energy (essentially, "the larger the potential energy, the larger the force") by finding three counterexamples, whether from the tutorial or from other homework problems, that refute the statement.
4. This problem, similar to Problem 3, forces students to critique an (incorrect) statement about force and potential energy (essentially, "the magnitude of the force is the same at every point along an equipotential contour") by finding a counterexample. By satisfactorily completing this problem and Problem 3, students should show that they understand what the gradient means and what it does not mean.