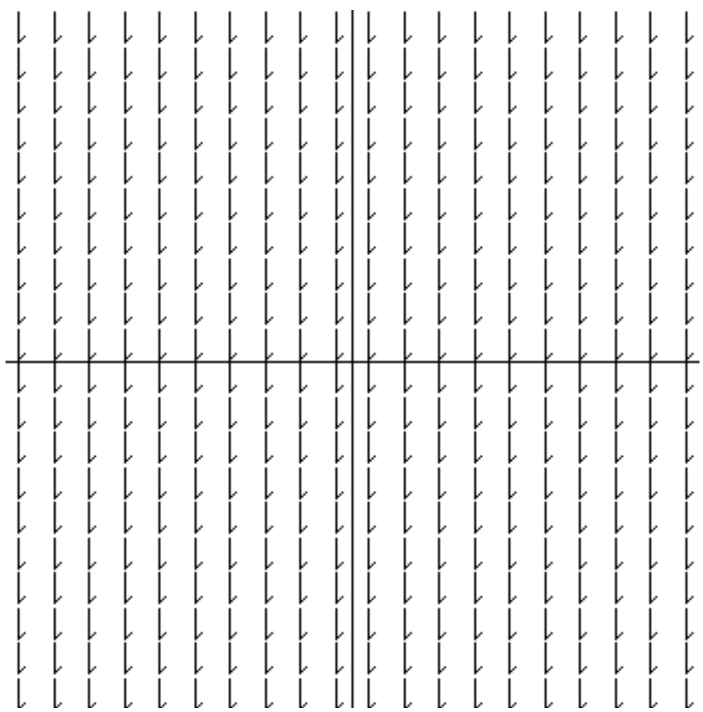
SEPARATING FORCES

I. Projectiles

Consider a simple projectile (no air resistance, constant weight, point particle) shot at a 45-degree angle with an initial speed of v_o. If you were to solve for the particle's maximum height or time of flight, you would break the motion into two coordinates, distance and height, and solve each somewhat independently.

A. Why is it ok to solve for the horizontal and vertical components separately? Spend no more than 3-5 minutes speculating with your group. We'll develop a more formal answer in this tutorial.

B. Below is a force field diagram for the projectile in part A.



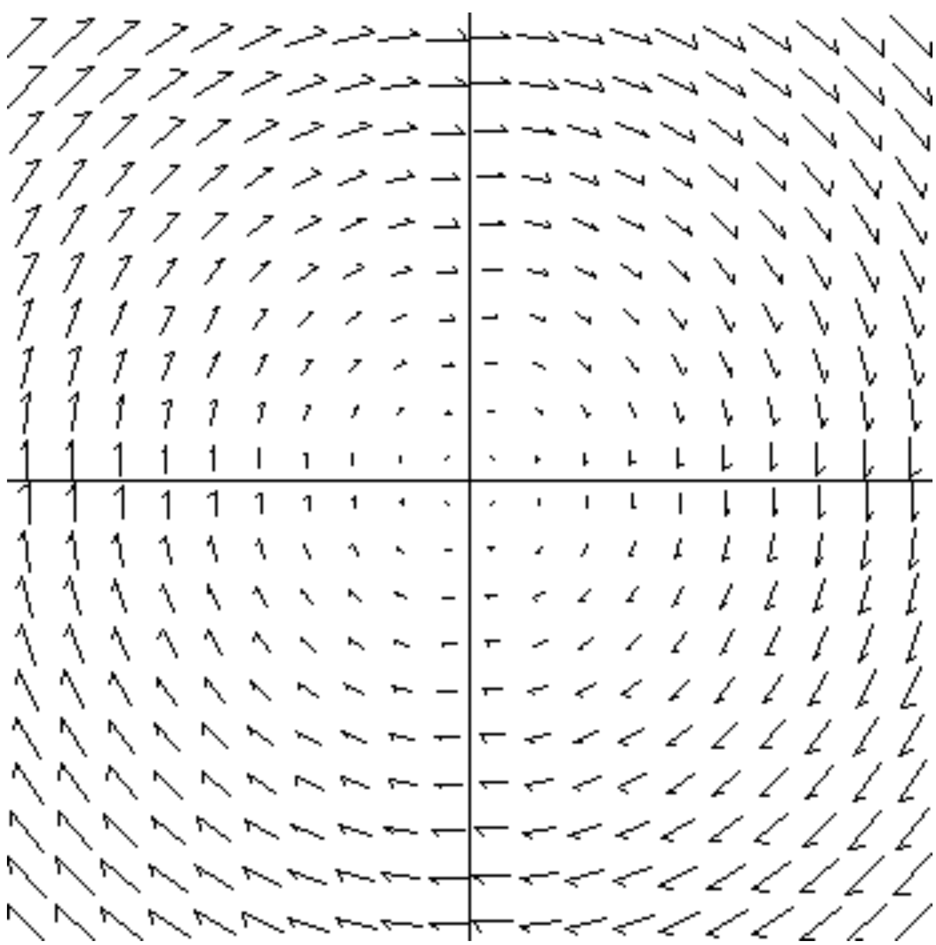
1. This field is conservative. Write how you can tell.

2. Does the amount of force in the vertical direction depend on the horizontal position? That is, as you scan the diagram from left to right, does the vertical component of the force change as you move horizontally?

3. Same question, but reversed: Does the amount of force in the horizontal direction depend on the vertical position?

4. If your answers to questions 2 and 3 are "no" and "no," then this force is separable in Cartesian coordinates: It can be divided into its horizontal and vertical components. In words, write a general rule for finding separability in Cartesian coordinates.

C. Below is a different force field. Is this force separable in Cartesian coordinates? Circle at least two places that help you decide.



II. Equations

Mathematically, we say that a force is separable in a given coordinate system when each of the six terms of the curl is identically zero – that is, when each term equals zero, not just when each component of the curl equals zero. You may recall that the curl of a force **F** in Cartesian coordinates is:

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

A. For the generic form above, write out each term of the determinant and set the expression for the curl equal to zero. You and your group mates should have the same answer.

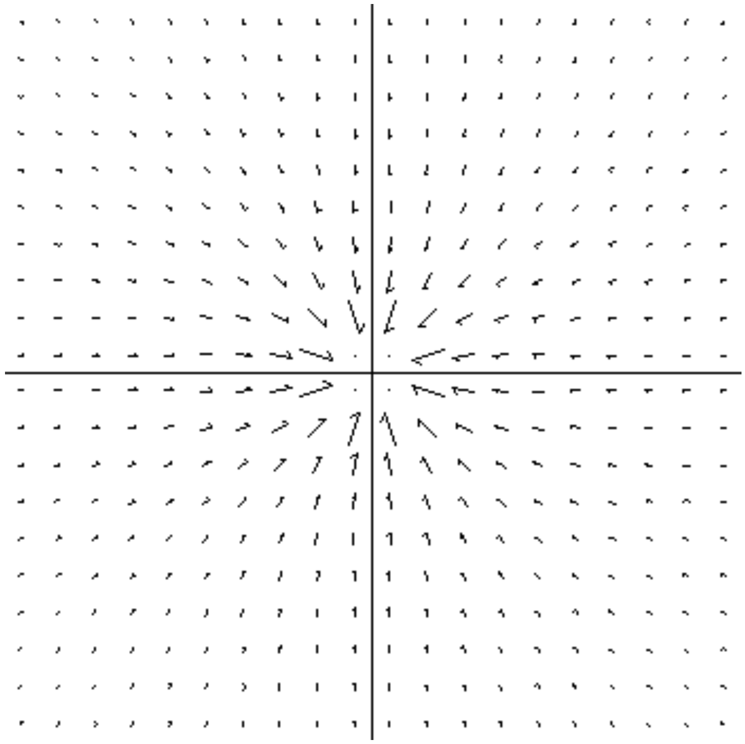
B. We said in I.A that the gravitational force $(\vec{F} = -mg\hat{k})$ was separable. Show this mathematically.

C. Which of the following forces are separable? Show your work in detail.

a) $\vec{F} = \alpha \cos(kx)\hat{i} + \beta\hat{j}$ b) $\vec{F} = \alpha y\hat{i} + \beta x\hat{j} + \gamma z\hat{k}$ c) $\vec{F} = \cos^2(ky)\hat{i} - \sin^2(kx)\hat{j} + \dfrac{\alpha}{z^2}\hat{k}$

III. Separability and conservation

A. The following field could have been drawn for the gravitational field of a planet.



1. Is this field conservative? Write how you can tell.

2. Is it separable in Cartesian? Circle at least two points that help you decide.

3. Is there another coordinate system for which it can be separated? (Hint: Yes.) Which one?

B. Is it possible to have the reverse: a field which is separable in Cartesian, but not conservative? If it is possible, write the equation for such a field below. If not, explain why not. In your answer refer to the definition of separability given in part II.

C. How, if at all, is separability different from conservation? Discuss with your group and the facilitators.

