### *Emphasis*

Students apply Newton's laws to various situations involving air resistance as a velocity-dependent force. The first section includes several qualitative applications. In the second section students are guided in the process of setting up appropriate differential equations for both upward and downward motion as well as calculating expressions for terminal speed.

### *Prerequisites*

It is recommended that students have had lecture instruction on air resistance prior to this tutorial. Familiarity with both the linear ($|F_{\text{air}}| = c_1 v$) and quadratic ($|F_{\text{air}}| = c_2 v^2$) formulations of air resistance is also helpful.

## TUTORIAL PRETEST

The first pretest problem probes the ability of students to draw and interpret free-body diagrams for a skydiver shortly after jumping from a plane and well after attaining terminal speed. Students are asked to compare the acceleration of the skydiver at both instants.

The second problem also asks the students to compare the acceleration of an object at two different instants of its motion. In this case the students consider the moment just before and the moment just after a rubber ball bounces off the floor. Unlike the first problem, the students are not explicitly asked to draw relevant free-body diagrams, which can make for an interesting comparison between those responses that do and do not include free-body diagrams. Many students incorrectly state that the accelerations are equal in magnitude. Some infer that because the speed of the ball is the same at both instants must mean that the accelerations are equal as well. Others seem to try to take into account the gravitational force and force of air resistance, but they fail to recognize or take into account the fact that air resistance switches directions from before to after the bounce.

## TUTORIAL SESSION

### *Equipment and handouts*

Each group will need a whiteboard and set of markers, or a large sheet of paper. Each student will need a copy of the tutorial handout (no special handouts required).

### *Discussion of tutorial worksheet*

*Section I: Air resistance and Newton's second law*

Part A of this first section gives students the chance to return to a situation similar to the skydiver problem from the pretest. Students draw free-body diagrams for a small ball soon after it is released from rest and well after it attains terminal speed. They then compare the acceleration of the ball at both instants. Students continue in part B by considering velocity versus time graphs for the ball with and without air resistance. Most students correctly represent terminal speed in their graph that takes into account air resistance. Furthermore, most correctly indicate that without air resistance the speed simply increases linearly with time. However, many fail to make both graphs consistent with each other; rather than make the slopes of the graph equal at $t = 0$, many draw graphs that cross one another. The checkpoint at the bottom of the first page allows instructors to make sure students draw correct graphs and can explain their answers thoroughly.

In part C students revisit the pretest problem about the rubber ball that bounces off the floor while moving under the influence of both gravity and air resistance. Students are guided to draw free-body diagrams for instants just before and just after the ball contacts the floor, and from their results they are expected to recognize that the acceleration of the ball is greater after the bounce than before. Students also critique two incorrect explanations that are often used on pretests to justify the notion that the accelerations are equal. In so doing those students who gave an incorrect explanation on the pretest have the opportunity to reflect on how their thinking has changed since then.

*Section II: Calculating terminal speed*

In this section students gain practice in setting up appropriate differential equations of motion for situations in which air resistance is treated as proportional to the speed of an object ($|F_{\text{air}}| = c_1 v$) and proportional to the square of the speed ($|F_{\text{air}}| = c_2 v^2$). In part A students are instructed to write down the differential equation of motion for an object moving vertically downward assuming that both the linear and quadratic expressions for air resistance are to be used. In part B, students write down the corresponding equation of motion for an object moving upward.

Students often need guidance in taking the direction of air resistance into account, as is evident from the rubber ball question on the pretest, and consequently many will have difficulty deciding which signs (whether "+" or "–") are appropriate for each term of the air resistance force. Taking downward as the positive direction, the students should obtain the following equations:

Downward velocity ($v > 0$): $F_{net} = m\ddot{x} = mg - c_1 v - c_2 v^2$

Upward velocity ($v < 0$): $F_{net} = m\ddot{x} = mg - c_1 v + c_2 v^2$

Students may not have much trouble recognizing that the linear air resistance term always carries the negative sign, in order to ensure that that term is always opposite in direction to the velocity. However many may overgeneralize this result to the quadratic term as well, forgetting that squaring the velocity removes any information about direction. Conversely, some students may write down the quadratic terms with their correct signs but make the signs on the linear terms always agree with those on the quadratic terms. These students may forget that $v$ is a signed quantity and thus carries information about direction. The instructor checkpoint at the end of part B is crucial for helping students to check their differential equations.

Finally, in part C students are guided to express the terminal speed of an object in a situation in which the force of air resistance is proportional to the speed (*i.e.,* $c_1 > 0$ and $c_2 = 0$). Students must apply the fact that motion at terminal speed implies zero acceleration and therefore $v_{\text{term}} = mg/c_1$. If time permits, encourage students to begin working on Problem 2 of the tutorial homework, in which they calculate terminal speed for other situations (see below).

**TUTORIAL HOMEWORK**

The homework gives students the opportunity to apply and extend their results from the tutorial, both qualitatively and quantitatively.

1. Students gain practice interpreting differential equations for objects moving upward or downward under the influence of air resistance. Taking upward as the positive direction (not downward, as done in tutorial), students should recognize whether each equation listed is possible for upward motion, downward motion, motion in either direction, or not at all.
2. Students extend their results from tutorial by calculating terminal speed for situations in which air resistance is expressed as (a) proportional to the square of the speed or (b) a combination of linear and quadratic terms.
3. Students apply the results from the tutorial and Problem 2 by determining whether the force of air resistance on a softball is best approximated using the linear or quadratic formulation.
4. Students repeat the process they used in the preceding problem for a falling oil droplet.