A General Framework for Incorporating Ethical Reasoning into Mathematical Modeling Problems

Feryal Alayont
Department of Mathematics
Grand Valley State University
Allendale, MI 49401, USA
alayontf@gvsu.edu
(616) 331-2302

Korana Burke
Department of Mathematics
University of California Davis
Davis, CA 95616, USA
kburke@ucdavis.edu
(530) 754-0821

Erin Griesenauer
Mathematics Discipline
Eckerd College
St. Petersburg, FL, 33711, USA
grieseel@eckerd.edu
(727) 864-8438

Jeremy A. Shaw
Department of Mathematics
Oregon State University-Cascades
Bend, OR, 97702, USA
shawjer@oregonstate.edu
(541) 706-2167

Rohit Thomas
Department of Mathematics
University of California Davis
Davis, CA 95616, USA
rpthomas@ucdavis.edu
(530) 754-0821

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Abstract: Ethical reasoning is an essential component of applying mathematical modeling in solving real-life problems, both in research and business settings. Our mathematical models exist in the context of a larger system and have implications on the lives of others, the planet, and future generations. However, mathematics instruction often treats mathematical work as if it exists in a vacuum devoid of context and omits careful consideration of affected parties, validity of data, assumptions made, and limitations of the analysis. In this article, we present a general framework that can be used to modify any mathematical modeling problem or project in a way that helps students to fill in the missing ethical reasoning components in the problem/project. This general framework can be applied to any course at any level, including K-12 instruction. It is designed with flexibility for instructors in terms of the types and depth of ethical questions that can be asked, as well as the quantity of questions. Multiple examples are included to illustrate the framework's application.

Keywords: ethical reasoning, mathematical modeling, critical thinking, ethics, context, reflection

1 Introduction

In recent years, there has been an increase in calls to incorporate ethics or ethical reasoning into the mathematics curriculum [10, 26, 1, 6, 30, 25]. Curriculum resources have also been developed to introduce these ideas through social justice topics [7, 17, 18, 33, 23]. This focus on ethics reflects the increasing recognition of the direct and immediate implications of mathematics in fields such as technology, data science, and artificial intelligence on individuals, humanity, nature, and all the creatures we share the Earth with [2, 22, 3]. It is crucial that mathematics users and practitioners are trained appropriately in the ethical use of mathematics, hence the proliferation of voices calling for greater accountability in mathematics education. In this paper, we propose a method accomplish ethics integration into the mathematics curriculum by incorporating ethical reasoning into existing mathematical modeling problems.

We argue that ethical reasoning is an intrinsic component of applying mathematics in any real-life context. Our students will inevitably use mathematics in their future careers and their actions will have consequences on all aspects of the world around us. Skovsmose makes this claim succinctly: "I find that the duty of mathematics education is not only to help students to learn certain forms of knowledge and techniques, but also to invite students to reflect on how these forms of knowledge and techniques might be brought in action." [29] Additionally, we claim that teaching ethical reasoning as a part of mathematical modeling narrows the gap between classroom math and how math is used in the real world. Furthermore, in order to build a good mathematical model, one has to include ethical reasoning: teaching mathematical modeling without ethical reasoning is in itself incomplete and unethical. Therefore, for all these reasons, we mathematics educators have a responsibility to both highlight the ethical reasoning component of mathematical modeling and to provide appropriate training in ethical reasoning skills to our students in mathematics courses.

Our paper starts by describing what we refer to by ethical reasoning and discuss how ethical reasoning appears in applications of mathematics. We then explain why we chose a framework method and list the benefits of this approach. After describing our framework in general, we demonstrate the use of the framework on multiple examples.

Before we begin, let us clarify what we mean by a framework as the word carries several potential meanings. In this paper, we are not presenting an ethical framework or a pedagogical framework. Instead, we use the more quotidian definition of a framework as a system or structure intended to guide a process. In this case, we are outlining a system of steps that can be used in the process of adapting existing examples and exercises used in mathematics classes by adding ethical reasoning components.

2 Ethical reasoning in math courses and mathematical modeling

Mathematics is inherently ethical when applied to real-life contexts. As Ernest notes [13], "Every use or application of mathematics, as a human activity in any practice, has ethical implications and can be judged ethically." Mathematical work always involves context, assumptions, objectives, limitations, and potential implications, all of which require ethical reasoning throughout the process. In [21], Oldfield even more strongly proposes that "The ethical part of modelling cannot be extracted from the technical process and indeed requires subject matter experts and interdisciplinary working for it to succeed. [...] the ethical process should be fully embedded into the modelling process, they are not separate."

If we consider the mathematical work our students will perform in their future careers and lives, the majority falls into categories of work that requires ethical reasoning, whether it be engineers designing buildings or financial analysts predicting futures. Yet, mathematics educators rarely, if ever, discuss the ethical reasoning involved in using mathematical tools students are learning in their classes. If we train students to select appropriate mathematical models, why not also teach them ethical reasoning as part of the process? As educators, it is our duty to prepare students for all aspects of their future responsibilities, including ethical considerations. Ernest clearly identifies this responsibility in his statement, "Mathematics teachers share the obligation to consider the ethical consequences of different pedagogies, and selections of content and representations of content" [12]. As mathematicians, we accept that writing is inherently part of doing mathematics and provide instruction in writing proper proofs and explanations. It is time to do the same in regards to another integral component of mathematical work: ethical reasoning. Additionally, as Paul and Elder argue, in teaching students critical thinking while disregarding the ethical component, we run "the risk of inadvertently fostering sophistic rather than fair-minded critical thinking" [24]. Finally, by removing any discussion of context and impacted parties from our classroom, we become complicit in the faulty perception common among mathematicians and our broader society about the purity and neutrality of math described in Ernest's quote: "Pure mathematics is viewed as neutral and value free, and therefore free of ethical responsibility. Applications of mathematics are seen as employing a neutral set of tools which, of themselves, are free from social responsibility" [13].

Before we continue further, it is necessary to clearly define ethical reasoning as the term often conjures the narrow meaning of 'doing the right thing.' In general, ethical reasoning is the practice of making decisions in a way that considers ethical principles and potential consequences of those decisions. In [32], the authors present a framework for ethical decision making that consists of five steps: recognizing ethical issues, gathering relevant facts for the decision, considering alternative solutions from multiple perspectives, making an ethical decision, and implementing and reflecting on the outcome.

Although background knowledge on ethics and various ethical lenses, laws, professional codes, and morality would be helpful in improving a student's ethical reasoning skills, we are not advocating teaching any of these topics in our mathematics courses. We assume students have access to this knowledge and focus instead on the application of ethical reasoning skills. That is, we focus on teaching ethical reasoning and not on teaching ethics.

In addition to fulfilling our ethical obligations as mathematics teachers, including ethical reasoning in our courses has important positive outcomes related to student motivation and interest. Some of the most common student complaints we all hear, especially in introductory math courses, are:

- What are the real-life applications of what we are learning?
- When am I ever going to use this in my career?

A common thread in these complaints is the desire to learn material that is relevant and applicable. Mathematical modeling questions–and, more broadly, application problems–provide students with the opportunity to engage with such material.

According to the GAIMME Report [14], one definition of mathematical modeling is: "Using math to explore and develop our understanding of real-world problems." The report further states that "A mathematical modeling question should force students to take ownership of some of the decisions along the way." While we recognize the distinction between mathematical modeling and application problems as outlined in the report, we intentionally take a broader approach in this paper. We use the term "modeling problems" to encompass both types. Specifically, we propose that while ethical reasoning is an inherent aspect of good mathematical modeling, it can also enhance application problems. By integrating ethical questions into application problems, we encourage students to make decisions, critically evaluate the implications of those decisions, and foster ownership and reflection–even in problem types that may not traditionally be classified as modeling problems.

Ethical reasoning can further help students connect modeling problems to situations that are both interesting and relevant to them. This helps create a personal connection to otherwise abstract problems, while prompting students to make decisions and take ownership of elements that might be neglected in the original statement of the problem. As a result, requiring ethical reasoning to be an everyday occurrence in our classrooms increases the educational value for students and underscores the relevance of mathematics.

Let us consider the steps in teaching mathematical modeling that also apply to application problems to some extent. We teach students to first establish a question they want to answer and to use that question to define the system. Next, students identify which parts of the system are important to consider and then use that knowledge to further simplify the problem so it can be studied using a given set of mathematical tools. Finally, students perform some kind of validation of the model by reflecting on the mathematical results. The only thing that tends to change in these steps from the first time students use math to solve a problem to the time they graduate from college is the sophistication of the mathematical tools used and of their reflections on the mathematical results. It is not surprising that in this context, students feel as though math is divorced from the real world or that connections made to the real world are made just so educators can avoid questions about relevance.

Each of the standard steps in mathematical modeling offers opportunities to integrate critical thinking and ethical reasoning, which complement each other. This approach enhances student engagement and highlights the relevance of math in real-world contexts, reinforcing its connection to everyday life.

When we ask students to determine the question that needs to be answered, it is natural to also ask them to reflect on who would benefit from their answer, who are all possible affected entities, and whether their question takes into account everything that is pertinent to the problem at hand. As students progress through the modeling and analysis, we can prompt them to consider whether their choices for including/excluding certain contributions to the model could lead to undesired consequences. Finally, during the validation step, it is natural to ask questions about who might be excluded from the data used to validate the model and what impacts that exclusion has on the model itself and on the excluded parties. All of these questions can and should be asked if one is to build a good mathematical model. Asking students to consider these questions when developing a model can give the model valuable meaning to a student. It can also give important footholds for students who are struggling with the mathematical techniques to stay engaged with the problem. This approach will become even more valuable by bringing personal and professional diverse perspectives through small group and whole discussions in the classroom.

This exercise can also show students that different professions have different interests about the same problem. For example, a civil engineer might be interested in finding the optimal location to place a bridge based on a completely different set of parameters than a city planner.

3 Why a Framework Format?

We propose a framework format for incorporating ethics into the mathematics curriculum through mathematical modeling problems and even more broadly, through application problems. The main idea of our framework is that ethical reasoning skills can be developed and assessed by adding ethical reasoning questions to any mathematical modeling problem or activity. Before we describe these questions in detail in the next section, let us mention the benefits of such an approach.

A key advantage of using a framework-based format is its adaptability across different courses and at multiple times within a course, provided that the courses incorporate real-world modeling problems. This helps us achieve the goal of "ethical reasoning across the curriculum," similar to the "writing across the curriculum" movement. We do not ask our students to explain their mathematical work in only one or two mathematics courses, but rather in every mathematics course. Similarly, if we want our students to develop agility and expertise in the ethical reasoning skills they will regularly use in their future careers and lives, we cannot expect them to develop those skills in a one-time workshop or even within one or two courses. Ethical reasoning should be included in multiple mathematics courses in the curriculum and repeatedly within each course. Consistent and regular use will help students develop and internalize these skills. In [9, 28, 15], authors describe a similar approach to integrating ethics into the computer science and machine learning curricula via ethics modules in multiple courses "to habituate students to thinking ethically" and advocate for such an approach.

Another reason for the framework format is to make the adoption of the ethical reasoning instruction and assessment easier for faculty. There are many reasons faculty might be hesitant to incorporating ethics into their mathematics courses. We adapt reasons listed in studies from computer science and engineering [31, 19, 5] to math and argue that hesitance in math likely arises from similar factors:

- perceived irrelevance of ethics to the math curriculum,
- an already overfull curriculum,
- faculty's lack of training or materials for integrating ethics,
- student resistance to ethics in math content classes,
- challenges in assessing ethics and adapting existing curriculum materials, and
- societal or institutional undervaluation of ethics instruction.

Our framework approach, where we advocate for making small modifications to existing problems in the curriculum, addresses most of these issues. Faculty do not need to create new material from scratch or make significant changes to existing material. They simply need to decide which prompts are appropriate for some of their homework problems or classroom activities and edit the prompts minimally if needed. Since the questions are asked in the context of math content, both faculty and students see the relevance of ethics to the content. In fact, with this approach students enjoy the math content more. Our approach does require additional class time allocated to discussion and assessment of these ethical reasoning questions. Especially during the first few times, students need modeling of appropriate responses and some guidance to discussing these topics. However, this additional time is minimal compared to alternative approaches such as discussing detailed case studies or having a dedicated ethics course in the curriculum, and the added student interest and engagement in the ethical discussions will make the extra time spent worth it. In addition, adding ethical reasoning questions to existing mathematical modeling problems is not restrictive in terms of class planning. One does not have to plan carefully where a new activity could fit, within a week or within a class period. It only requires some extra time to discuss the added ethical reasoning questions and guide students in answering them appropriately.

4 Description of the Framework

Our framework outlines a general process for adding ethical reasoning questions to existing mathematical modeling activities or exercises. Similar ethical questions often already exist in upper-level applied math classes, such as mathematical biology or mathematical modeling courses. However, our framework can be used in lower-level courses to integrate ethical reasoning into application problems or activities where it is currently underemphasized. Any framework for this purpose must take into account the general structure of modeling problems themselves. We assert that mathematical modeling problems can be split into three broad parts. The first step consists of gathering background information and understanding the context in which the model will be used. The second step consists of the mathematical development of the model. The third step is reflecting on and evaluating the results of the mathematical work.

For the majority of application problems available in popular textbooks, students are typically not asked to consider the assumptions and context of the model, nor to reflect on the consequences of using the model. That is, throughout many of their classes, students are only exposed to the second step of modeling. Students may encounter a few problems that consider the context and consequences of the model, but in large part, most problems only introduce the context in a few sentences and then ask mathematical questions related to using the model. Students thus get an incomplete picture of what it means to model mathematically. Our framework helps elevate the oft-neglected first and third steps, suggesting to students that real-world context and consequences are just as important as calculations. We provide a simple approach to adapting existing questions that only consider mathematical computations by adding questions that address context and consequences.

This framework for viewing mathematical modeling problems in three parts allows instructors to identify which parts are missing from an existing problem and to write some questions to add to the problem. It is entirely up to the instructor how many ethical reasoning questions to add to a problem and how many categories of questions to add. The framework gives instructors the flexibility to customize modeling problems to include ethical reasoning in a way that fits with their lesson plans.

Modeling problems written using this framework can be designed for any assessment setting, including lecture examples, class group work, homework, and exams. Instructors who are beginning to bring ethical reasoning into their courses might wish to use this extended modeling framework just a few times during a semester, or instructors could build it into many of their assignments and activities. We suggest that the framework be used repeatedly throughout a course in various settings and with multiple topics so that students themselves learn to ask these questions each time they consider a model and are comfortable answering them on an exam. This framework is general enough to be incorporated into any course that uses modeling, and departments might want to structure course sequences to build up the skill of ethical reasoning, for example by starting with a few examples in Precalculus and adding more through Calculus and Differential Equations courses.

For each of the three stages of model development, there are many potential questions to add. During the first stage of collecting background information and understanding the context in which a model will be used, potential questions include:

- List two or more Business-Industry-Nonprofit-Government (BING) organizations who might use this model, and what questions they might be trying to answer.

- What parameters will be/are included in the model? How are these parameters estimated? Are there other potential parameters that are left out of the model?

- What data are used? How was that data collected?

- What trade-offs are made when choosing which type of model to use or how to collect data? How will these choices impact the results? Who might be impacted by these choices?

The second phase of model development involves performing mathematical computations using a model. This step is where much of the focus in teaching mathematical models resides currently. This is especially true in earlier courses such as Calculus, where the emphasis is on learning mathematical techniques and applying them to models that have already been developed, rather than developing the models themselves. However, we see an opportunity here to add more questions that focus on students' understanding of the model and how it is related to the real-world phenomenon it is modeling. In this stage, questions that might be asked include:

- What type of model is being used? What assumptions are being made? Are those assumptions reasonable?

- What are the constants in the model? Where did they come from and what do they represent?

- What parameters are included in the model? What is the physical interpretation of each parameter? What can you say about the possible values of these parameters and why?

The third phase of model development is to assess the predictions made by the model and reflect on the decisions that might be made based on these predictions. In assessing the predictions made by the model, we might ask students the following:

- How accurate does the model seem? Do the predictions match your intuition? If not, can you identify why?

- Does the model overestimate or underestimate risk?

- How might you assess the accuracy of the model?

In reflecting on the decisions that might be made by this model, questions that might be asked include:

- What entities might be harmed or might benefit from this model?

- What policy decisions might be made based on this model?

- Name three other questions that could be answered using this model.

This is a long list of questions, and instructors are unlikely to use all of them for any specific modeling example. Instead, we recommend adding a few to the modeling examples or problems already used in class. Even adding just one or two can deepen students' understanding and encourage them to consider the model in its physical, social, and real-world context rather than as an abstract mathematical object.

This list of questions is also not exhaustive. These questions can be combined or extended to make new questions, and they can be tailored and adapted to fit specific models. In the next section, we will look at examples of how these general questions were adapted to be added to specific models that the authors use in their classes.

As an alternative to the large variety of ethical reasoning questions an instructor must write or choose from, we offer the following simpler version:

- Consider the context of this problem. What assumptions are being made in this problem regarding the real-life context? How can we verify the reasonableness of these assumptions? List some possible parties that might be affected positively/negatively by these assumptions and indicate how they might be affected. What conflicts of interest do you notice?

This single prompt can be added at the end of any modeling problem to incorporate ethical reasoning. Using this single question does not require thinking about each modeling problem separately to write appropriate specific questions. The wording in this prompt was chosen specifically to encompass both the background/context and reflection parts of the framework in one question which can be applied to all modeling problems. The prompt can be used repeatedly throughout a course and students will become accustomed to thinking about how their work impacts their professions, their reputations and those of their colleagues, other people, and animate and inanimate beings more generally.

One challenge that might occur is when a mathematical modeling problem, especially in a lower-level course, does not have enough context for students to successfully answer the questions about the context. In that case, a slight modification of the prompt can be used to make the problem come alive. The first sentence can be replaced by 'Consider a potential real-life context for this problem without changing the parameters already given.' This prompt lets students think about a context for the modeling problem of their choice. This option gives students agency and imagination in addition to the chance for ethical reasoning reflection.

Our framework is similar to frameworks in data science and machine learning, considered by others. The Ethical Decision-Making in Data Science Protocol developed by Register et al. described in [27] contains multiple guiding questions in specific categories "intended to promote [sociopolitical knowledge], [data science knowledge], as well as other forms of knowledge, and a pluralistic moral disposition including concern for social responsibility and an ethic of care." In [28], a list of "Ethical Questions About Machine Learning" were presented as "foundational ethical questions ... focused on helping students understand the potential ethical conundrums that might be encountered within [a machine learning] project." Both lists of questions are similar to our list in purpose, to help students think critically about ethical considerations related to a given problem.

5 Demonstration of the Framework

To demonstrate how to use the framework, we show three mathematical modeling problems that were modified to include ethical reasoning. This first modeling problem was given to students in an assessment in a recent calculus class.

> Whooping cranes are an endangered species. Below are data on the number of wild whooping cranes each decade from 1940 to 2010, taken from [8].
>
> | Year | 1940 | 1950 | 1960 | 1970 | 1980 | 1990 | 2000 | 2010 |
> |---|---|---|---|---|---|---|---|---|
> | Cranes | 26 | 31 | 36 | 57 | 78 | 146 | 180 | 283 |
>
> (a) Make linear, log-log, and log-linear plots of the data.
>
> (b) Determine which plot shows a relationship closest to linear. Find this best-fit line and use it to find the functional relationship between population and time.

This problem provides only a brief mention of whooping cranes and their population over time. If we were to add ethical reasoning to this problem, there are many questions that can be added. A number of such questions relating to the background and model validation are listed below.

- How would you test the accuracy of this model?
- Do you think the model is reliable in its predictions of the crane population over very short intervals?
- Are there reasons to suspect that the growth rate of the population might not be constant? *Hint:* Whooping cranes have a very specific breeding season.
- What challenges do you foresee in the process of estimating the population numbers? What potential trade-offs might be made when estimating these numbers?
- Could you predict the number of cranes 10 years in the future? 100 years? Would you have confidence in these predictions?

Next are a sampling of questions that can be added reflecting on the model. These, in particular, focus on analyzing the parties affected by the use of this population model.

- List at least two different Business, Industry, Non-profit, or Government organizations who might use this model. For each, list at least one question they might be trying to answer.
- What are possible consequences if this model is inaccurate? What entities might be affected?
- What kinds of policy decisions might be made using your results? What entities might be affected by these decisions?
- From the perspective of an organization using this model, is it better to overestimate or underestimate risks? (What risks are there to think about?) How would you revise your model in order to do this?

- What further questions could you investigate with this model? What are some potential benefits and harms that could come from this model?

As we acknowledged in the previous section, these are far too many questions to include in this problem. We provide this large number of potential questions to show the kinds of questions that can be asked. An instructor may choose one or two (or more) for each portion to ask or choose one to two overall to add. Depending on the time available or how involved an instructor would like the discussion to be, they may decide to include more or fewer questions.

The next example [20] is relevant for a differential equations course.

> The secretion of hormones into the blood is often a periodic activity. If a hormone is secreted on a 24-h cycle, then the rate of change of the level of the hormone in the blood may be represented by the initial value problem
>
> $$\frac{dx}{dt} = \alpha - \beta \cos \frac{\pi t}{12} - kt, \qquad x(0) = x_0,$$
>
> where $x(t)$ is the amount of the hormone in the blood at time t, α is the average secretion rate, β is the amount of daily variation in the secretion, and k is a positive constant reflecting the rate at which the body remove the hormone from the blood. If $\alpha = \beta = 1$, $k = 2$, and $x_0 = 10$, solve for $x(t)$.

This problem for modeling hormone secretion in the body is lacking the context for a specific situation where this model applies. Below are some questions that can be included to expand this problem.

- What are some examples in which applying this model is valuable? (Possible examples include thyroid level monitoring, estrogen/testosterone monitoring, and insulin monitoring.)
- What units are appropriate for each of the model parameters (α, β, and k)? How might you estimate them?
- Would the parameters likely be the same for everyone? Why or why not?
- Does this work for different body sizes? What about kids or infants?
- What are the potential harms if this model is used to prescribe inaccurate dosages?

While there are not as many new example ethical reasoning prompts for this example as there were for the whooping cranes example, ethical reasoning questions can be adapted and reused in multiple examples. When this happens, instructors will help students develop the habit of thinking more broadly about a problem whenever they encounter them. We encourage instructors who use this framework to reuse their work in different problems as opposed to creating new ethical reasoning prompts for every problem.

To further demonstrate this framework, one additional example [4] is shown, which is relevant to derivatives in a calculus course.

> City planners model the size of their city using the function
>
> $$A(t) = -\frac{1}{50}t^2 + 2t + 20, \quad 0 \le t \le 50,$$
>
> where A is measured in square miles and t is the number of years after 2010.

(a) Compute $A'(t)$. What units are associated with this derivative and what does the derivative measure?

(b) How fast will the city be growing when it reaches a size of 38 square miles?

(c) Suppose the population density of the city remains constant from year to year at 1000 people per square mile. Determine the growth rate of the population in 2030.

This problem already includes some ethical reasoning through part (a), whereby it asks about the units and the context of the derivative. So, in this case, the simpler one-part prompt works as a good addition to further expand upon the ethical reasoning.

- Consider the context of this problem. What assumptions are being made in this problem regarding the real-life context? How can we verify the reasonableness of these assumptions? List some possible parties that might be affected positively/negatively by these assumptions and indicate how they might be affected. What conflicts of interest do you notice?

The examples in this section illustrate how ethical reasoning can be incorporated into existing problems. Instructors have the flexibility to decide how much emphasis to place on ethical reasoning and what type of questions to include. Although the framework is demonstrated using foundational mathematics courses, it is equally applicable to other types of courses, such as mathematical biology, statistics, and cryptography, to name a few.

6 Student Responses and Feedback

In a recent calculus class, ethical reasoning questions from our framework were used once a week. Students completed pre- and post-test surveys. This study was approved by the University of California Davis institutional review board (approval number 2105781-1). Informed consent was obtained from all participants.

When the whooping crane population problem from section 5 was used in this class, the instructor added the following prompts:

(c) Who might be interested in your predictions [of crane populations], and for what purpose?

(d) If your predictions were too high/low, what would be the consequences for the people using your predictions?

Common student responses to part (c) included conservationists, ecologists, zoologists, and wildlife protection organizations, all using predictions to try and save the endangered cranes. Many students expanded on this in their responses to part (d), noting that these predictions could be used to determine whether resources being used to stabilize crane populations were sufficient or might be better used on other endangered species.

Out of all students in the course, 81% completed pre- and post-surveys. Two of the survey questions and their response statistics are below.

- "I am prepared to think about who my decisions will affect (and how)."

  - 94% of respondents agree!
  - Of these 94%,
    * 34% agreed *more* than at the beginning of the course,
    * 60% agreed the same amount, and
    * 6% agreed *less*.
- "In the future it will be important to think about who my decisions will affect (and how)."
  - 92% of respondents agree!
  - Of these 92%,
    * 36% agreed *more* than at the beginning of the course,
    * 61% agreed the same amount, and
    * 3% agreed *less*.

The instructor for this course reported that while the inclusion of ethical reasoning questions did not increase students' enjoyment of calculus or their computational skills, it did significantly increase students' perception of the value of calculus in their future careers.

The simpler model of adding the generic ethical reasoning questions at the end of select mathematical modeling problems was used in a single-variable calculus course in Fall 2023. Throughout the semester, students completed a total of five such problems, four of them during class activities and one in a homework assignment. At the end of the semester, students completed a portfolio assignment assessing their growth on the course objectives. One of the objectives focused on ethical reasoning. In their responses, many students commented that this was their first time thinking about ethical implications of mathematical work. One student comment, used with permission, specifically highlights the lack of ethical discussions in the math courses despite the significant ethical implications of our work.

> This class is my first experience with viewing math through an ethical perspective. In high school, all of the math problems we solved were, well... problems. It was 'someone needs you to calculate this thing to help them' or 'how many of these would you need'. It was never a question of should we solve this problem.
>
> An engineering class in high school introduced me to moral dilemmas associated with problem solving, especially in terms of providing companies with products that could be used to do wrong, but I'd never really thought about it in terms of math. Solving problems in a way that minimizes negative effects to others is extremely important to me, especially as somebody going into AI.

It is especially striking to note that the student was exposed to ethical reasoning in high school already in an engineering course content. Yet we math instructors rarely, if ever, discuss any ethical reasoning in our courses even at the college level.

Despite the minimal inclusion of the ethical reasoning discussion into the course content, most of the students indicated that the exercises helped them to think critically about their mathematical work and to become aware of how their work may impact others. These discussions also allowed students to become more engaged and interested in the material, most likely due to the added context helping the problems come alive.

7 Conclusion

We believe that ethical reasoning instruction is an essential part of teaching mathematical modeling. Our framework provides a simple way for instructors to incorporate these discussions into their existing course materials, enriching the landscape of ethics in mathematics. Early implementations of our framework have shown positive results for both students and instructors. We encourage instructors to explore mathematical modeling as a natural entry point for introducing ethics, as it can seamlessly integrate into math and math-adjacent courses. Additionally, our role as instructors includes preparing students for their future professional lives and, as Gunsalus argues in [16], who can better prepare students for ethical professionalism in our field than ourselves?

There remains work to be done to improve our framework. Our framework can be developed further into a complete protocol similar to the Ethical Decision-Making in Data Science Protocol [27]. We invite readers to use our framework, modify it, and suggest improvements. Additionally, we have not focused much on assessment, both of the framework itself and of the student answers to the added ethical reasoning questions. Larger and more comprehensive studies can be designed to see if students' ethical reasoning skills can be measured and improved after ethical reasoning is emphasized in the classroom, perhaps through interviews of focus groups of students. In our applications of the framework in our classes, the ethical reasoning questions we considered mainly focused on awareness and reflection aspects of the ethical reasoning skills. We have not focused much on the "considering alternative solutions from multiple perspectives, making an ethical decision, and implementing and reflecting on the outcome" dimensions of ethical reasoning. Curriculum materials that involve these aspects in a detailed manner would be helpful, although are most likely to be beyond the scope of a single problem.

Another potential direction that can be taken is proposed by de Freitas in [11]: revising problems that already exist to include realistic ethical dimensions. Specifically, de Freitas gives her students "a set of 'real' life applications and ask[s] to re-write these in ways that address 'citizenship, environmentalism or social responsibility.'" In [11], the goal was for future educators to learn how to make the problems their students would solve more connected to real-life from an ethical perspective. In our case, such an exercise would give students more agency in determining the direction of the problem and potentially allow for more genuine ethical considerations as students will be able to notice the issues better in a topic they care about. In their future careers, they will be expected to rewrite problems they are given to incorporate ethical reasoning and the best way to learn to do that is by practicing now.

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Biographical Sketches

Feryal Alayont received her undergraduate mathematics degree from Bilkent University, Türkiye, and her Ph.D. in mathematics from the University of Minnesota, USA. She is currently a professor of mathematics at Grand Valley State University, USA.

Korana Burke received her undergraduate mathematics degree from the University of California Berkeley and her Ph.D. in physics and chemistry from the University of California Merced. She is currently a continuing lecturer at University of California Davis.

Erin Griesenauer received her undergraduate mathematics degree from the University of Tulsa and her Ph.D. in mathematics from the University of Iowa. She is currently an associate professor of Mathematics at Eckerd College.

Jeremy Shaw received his undergraduate mathematics degrees and Ph.D. in Mathematical Sciences at Portland State University in 2017. He is currently a senior instructor at Oregon State University-Cascades.

Rohit Thomas received his undergraduate mathematics degree from Caltech and his Ph.D. in mathematics from the University of California Davis. He is currently a continuing lecturer at University of California Davis.