ACCELERATING REFERENCE FRAMES: THE FOUCAULT PENDULUM

Recall the four types of fictitious "forces" for non-inertial reference frames, listed below. [*Note:* The primed quantities describe the motion of an object with respect to a non-inertial frame, while $\vec{A}$, $\vec{\omega}$, and $\dot{\vec{\omega}}$ describe the motion of the non-inertial frame with respect to an inertial frame.]

Inertial "force": $\vec{F}'_{inertial} = -m\vec{A}$

Coriolis "force": $\vec{F}'_{Coriolis} = -2m\vec{\omega}\times\dot{\vec{r}}'$

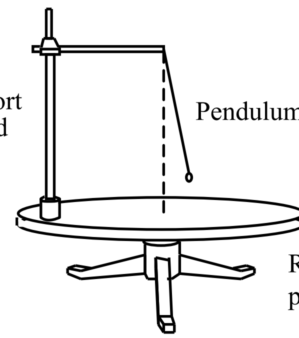
Centrifugal "force": $\vec{F}'_{centrifugal} = -m\vec{\omega}\times(\vec{\omega}\times\vec{r}')$

Transverse "force": $\vec{F}'_{transverse} = -m\dot{\vec{\omega}}\times\vec{r}'$

As you work through this tutorial, ignore the effect of the Earth's rotation unless otherwise specified.

I. Pendulum observed in a rotating frame

A tall simple pendulum oscillates back and forth in a vertical plane. Imagine observing the pendulum while standing underneath it on a large rotating platform. The pendulum is suspended from a point directly above the rotation axis of the platform. (The apparatus shown at right may help you visualize this situation.)



Suppose that, as viewed from above, the platform were to rotate counter-clockwise at a constant rate. Also, assume that the pendulum swings back and forth many times during one complete rotation of the platform.

A. In answering the following questions, imagine that you are an observer at rest with respect to the platform.

1. Describe in words how the plane of oscillation of the pendulum would gradually change orientation (or *precess*) as observed in the platform frame.

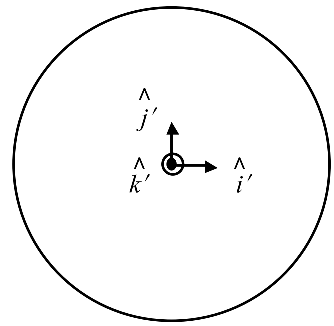
   How would your answer be different if the platform were rotating *clockwise* (rather than counter-clockwise) relative to the lab frame?

2. When viewing the pendulum bob in the platform frame, describe how the pendulum bob appears to be deflected with each swing. (That is, does the pendulum bob "turn left," "turn right," or does it "turn" both ways?)

3. How does the period of precession of the pendulum (not the period of oscillation!) compare to the period of rotation of the platform (as observed in the lab frame)?

B. The observed deflection of the pendulum in the platform frame can be explained by one of the four fictitious "forces" in that frame. Which fictitious "force" is it?

Top view diagram of platform frame
*(Platform spins **CCW** in lab frame)*



Explain your reasoning in words and diagrams, and in your explanation account for the way in which the pendulum bob would be deflected if it were to pass through the equilibrium position:

- moving in the $+\hat{i}'$ direction

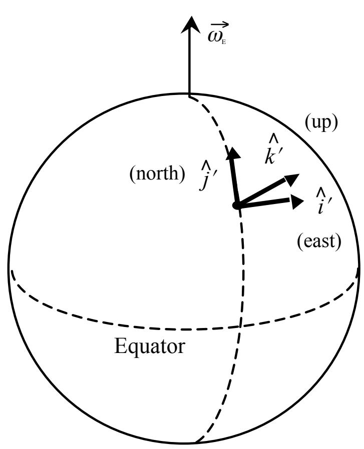
- moving in the $+\hat{j}'$ direction

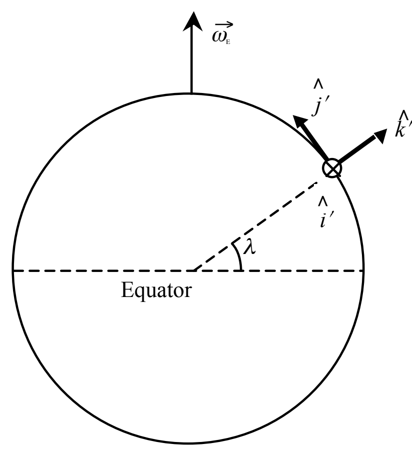
✓ **STOP HERE** and check your results with an instructor before continuing.

II. Pendulum observed on a rotating Earth

We will now consider the effect of the rotational motion of the Earth on a simple pendulum. The diagrams below illustrate a coordinate system whose origin remains fixed at latitude λ and whose axes coincide with local vertical (*i.e.,* pointing opposite the local acceleration due to gravity), north, and east.

(*Note:* Despite the fact that the Earth is not exactly spherical in shape, we will approximate the latitude λ as being measured from the exact center of the Earth.)





A. Consider first the case in which the pendulum oscillates (with small amplitude oscillations) directly above the North Pole (*i.e.,* $\lambda = 90°$).

1. With your partners, extend your reasoning from section I to determine whether the pendulum precesses *clockwise* or *counter-clockwise* according to an observer standing at the North Pole.

2. Would the period of precession of the pendulum be *longer than, shorter than,* or *equal to* 24 hours (the period of rotation of the Earth)? Explain.

B. Now compare the pendulum at the North Pole to another (identical) pendulum oscillating at a location of arbitrary northern latitude $\lambda = +\lambda_o$. This new pendulum will also experience precession.

1. Would the new pendulum precess *clockwise* or *counter-clockwise* (when viewed from above the point of support)? Defend your answer by applying your knowledge of the appropriate fictitious "force" to each of the following possible cases:

   - The pendulum is set into motion with an initial <u>northward</u> ($+\hat{j}'$) velocity.

   - The pendulum is set into motion with an initial <u>eastward</u> ($+\hat{i}'$) velocity.

2. For each of the two cases you considered in part 1 above, show that the *horizontal component* of the appropriate fictitious "force" at latitude $+\lambda_o$ is never as large as it is at the North Pole.

   [*Hint:* For each case, first consider the *magnitude* of the appropriate "force" itself—perhaps the force itself won't be as large. Then, if necessary, consider the *horizontal* component of that "force," *i.e.,* the component that points tangent to the Earth's surface.]

   Your results here in part 2 lead to an important conclusion: At latitude $+\lambda_o$, would the plane of oscillation precess *more quickly than, more slowly than,* or *at the same rate as* at the North Pole? Explain.

✓ **STOP HERE** and check your results with an instructor.

A simple pendulum undergoing small amplitude oscillations that is viewed in an Earthbound reference frame is often called a *Foucault pendulum* or *spherical pendulum.*

C. Summarize and extend your results: For a Foucault pendulum at each of the locations listed below, compare the direction (whether clockwise or counter-clockwise) and rate of precession with those of a pendulum located at northern latitude $+\lambda_o$. Discuss your reasoning with your partners.

- at a location slightly farther north *(i.e.,* at a new northern latitude $\lambda > +\lambda_o$)

- at the equator (*i.e.,* $\lambda = 0°$)

- at the south pole (*i.e.,* $\lambda = -90°$)

- at southern latitude $\lambda = -\lambda_o$