Workshop W26:

Intermediate Mechanics Tutorials

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Outline of workshop activities

- Introductions (20 min)
- Topic #1: Damped oscillations
  - Sample tutorials (45 min each)

    Damped harmonic oscillations: Motion graphs

    Graphical analysis of differential equations
  - Discussion of student reasoning (25 min)
- Break (10 min)
- Topic #2: Conservative force fields
  - Sample tutorial: *Conservative forces and equipotential diagrams* (45 min)
  - Discussion of student reasoning (25 min)
- Concluding discussion and exit survey (20 min)

Fundamental assumptions

- A major goal of physics instruction is the development of the ability to correctly *analyze*, *model*, and *predict the outcome of* physical phenomena.

- Both qualitative and quantitative reasoning skills are essential.

General questions for research

- To what extent is the ability to correctly analyze, model, and predict physical phenomena an outcome of instruction?
- What are some debilitating hurdles that learners encounter?
- Which instructional strategies are productive (and under what circumstances)?

From previous research at the introductory level

After standard lecture instruction in introductory physics, most students:*

– lack a *functional understanding* of many basic physical concepts

(*i.e.*, they lack the ability to apply a concept in a context different from that in which the concept was introduced)

– lack a coherent framework relating those concepts

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* McDermott and Redish, "Resource letter PER-1: Physics Education Research," Am. J. Phys. **67** (1999).

What is “intermediate mechanics” about?

Review of fundamental topics

- Vectors
- Kinematics
- Newton’s laws
- Work, energy, energy conservation
- Linear and angular momentum

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New applications and extensions

- Velocity-dependent forces
- Linear and non-linear oscillations
- Conservative force fields
- Non-inertial reference frames
- Central forces, Kepler’s laws

New formalism and representations

- Scalar and vector fields; del operator; gradient, curl
- Phase space diagrams

As an *instructor* of intermediate mechanics

One might expect students to have already developed:

- *functional understanding* of physical concepts covered at the introductory level

- mathematical and reasoning skills necessary to extend those concepts in solving more sophisticated problems, *both qualitative and quantitative*

As a *physics education researcher* teaching intermediate mechanics

The following research questions arise:

- To what extent have students developed a functional understanding of fundamental concepts in mechanics?
- What difficulties arise when students encounter topics in intermediate mechanics? To what extent do those difficulties have their roots in fundamental concepts?

Challenges of physics education research on advanced topics

Types of research instruments:

- Multiple-choice
- Multiple-choice, multiple-response
- ▷ Constructed response
  - *"Explain your reasoning."*
- ▷ Informal observations of students in and out of classroom
- Structured interviews

Challenges of physics education research on advanced topics

Types of research tasks:

- Definitions of terms or concepts
- Descriptions of phenomena in own words
- Standard quantitative (end-of-chapter) problems
- Context-rich problems
- Qualitative predictions, comparisons, rankings

Challenges of physics education research on advanced topics

Measurements: student responses

Inferences: student reasoning, beliefs, expectations

Given a research instrument or task, how can we tell (if at all) whether students proceed:

- from reasoning → to response?
- from response → to justification?

Context of investigation and curriculum development

Primary student populations: Intermediate mechanics

- Grand Valley State University (GVSU)
- University of Maine (UME)
- Seattle Pacific University (SPU)

Primary research methods

- Ungraded quizzes (pretests)
- Written examinations

} ***"Explain your reasoning."***

- Informal observations in classroom

A tutorial approach for teaching intermediate mechanics

- Modeled after *Tutorials in Introductory Physics* (McDermott, Shaffer, and the P.E.G. at U. Washington)
- Emphasis is on:
  - developing conceptual understanding and reasoning skills
  - making sense of mathematical formalism
- Tutorial components:
  - pretests (ungraded quizzes, in-class or take-home; 10 min)
  - tutorial worksheets (small-group work; 40 – 50 min)
  - tutorial homework
  - examination questions (post-tests)

Intermediate Mechanics Tutorials

Collaboration between GVSU (Ambrose)* and U. Maine (M. Wittmann)

- Simple harmonic motion
- Newton's laws and velocity-dependent forces
- Damped harmonic motion
- Driven harmonic motion
- Phase space diagrams
- Conservative force fields
- Harmonic motion in two dimensions
- Accelerating reference frames
- Orbital mechanics
- Generalized coordinates and Lagrangian mechanics

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* Ambrose, "Investigating student understanding in intermediate mechanics: Identifying the need for a tutorial approach to instruction," *Am. J. Phys.* **72** (2004).

Topic #1

Damped harmonic oscillations

What we teach about harmonic oscillations

in intermediate mechanics

	Equation of motion	Solution for *x(t)*
Simple harmonic motion	$m\ddot{x} = -kx$	$x(t) = A_o \cos(w_o t + j)$ where $w_o = \sqrt{k/m}$
Underdamped motion	$m\ddot{x} = -kx - c\dot{x}$ (or, $\ddot{x} = -w_o^2 x - 2g\dot{x}$)	$x(t) = A_o e^{-gt} \cos(w_d t + j)$ where $w_d = \sqrt{w_o^2 - g^2}$

$\Rightarrow$ Frequency depends on mass and spring constant

$\Rightarrow$ Amplitude has no effect on frequency

What we teach about harmonic oscillations

in intermediate mechanics

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Simple harmonic motion	$m\ddot{x} = -kx$	$x(t) = A_o \cos(w_o t + j)$ where $w_o = \sqrt{k/m}$
Underdamped motion	$m\ddot{x} = -kx - c\dot{x}$ (or, $\ddot{x} = -w_o^2 x - 2g\dot{x}$)	$x(t) = \boxed{A_o e^{-gt}} \cos(w_d t + j)$ where $\boxed{w_d = \sqrt{w_o^2 - g^2}}$

$\Rightarrow$ Amplitude decreases as time elapses

$\Rightarrow$ Damping force lowers oscillation frequency ($w_d < w_o$)

Research questions: Student understanding of oscillations

After relevant lecture instruction:

- How well do students understand the factors that affect the **frequency** of oscillations?
  - Simple harmonic motion in 1-D and 2-D
  - Damped harmonic motion
- How well do students relate the physics of oscillators to **formal representations** of their motion?
  - Motion graphs of 1D oscillators
  - Differential equations of motion
  - Phase space diagrams of 1D oscillators
  - Real space *(x-y)* trajectories of 2D oscillators

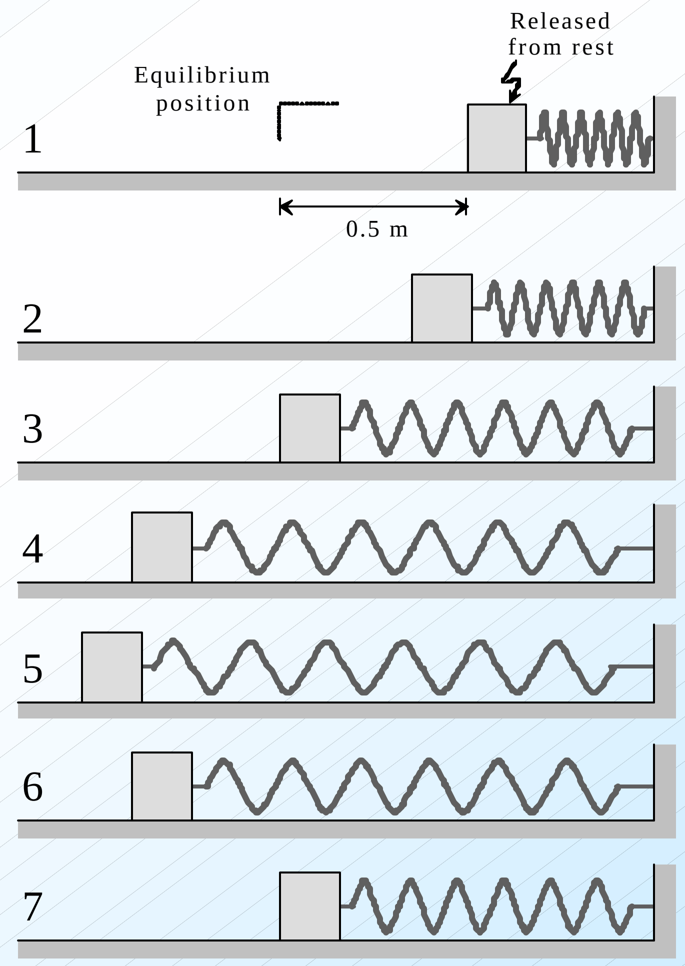
“Simple harmonic oscillator” pretest

(excerpt)

A block is connected to a spring and placed on a frictionless surface. A student releases the block 0.5 m to the right of equilibrium.

For each change listed below, how (if at all) would that change affect the **period** of motion? Explain your reasoning.

- The block is released 0.7 m to the left of equilibrium.
- The spring is replaced with a stiffer spring.
- The block is replaced with another block four times the mass as the original one.



Simple harmonic oscillator pretest: Results

Intermediate mechanics, GVSU (4 classes) and SPU (1 class)

***After** relevant lecture instruction*

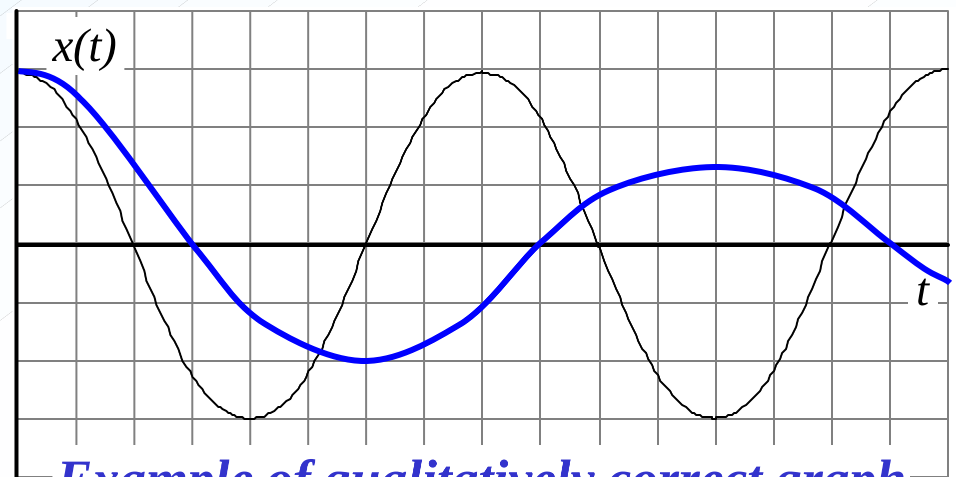
Predicting the effect of **increasing the initial displacement:**

- Most students correctly predicted no change to the period, *however* very few (~ 10%) gave complete explanations.
- Incorrect responses seemed to be based on *only one* of several (often competing) factors:
  - *Distance traveled by block:* "[Period is] larger, because the block travels farther during each period."
  - *Acceleration of block:* "Acceleration more, thus moving faster, so period is shorter."

“Underdamped oscillator” pretest

Part i

The *x* vs. *t* graph represents the motion of a simple harmonic oscillator that is released from rest at $t = 0$.



Example of qualitatively correct graph

A. Clearly indicate and label (i) amplitude, (ii) period. Explain your reasoning.

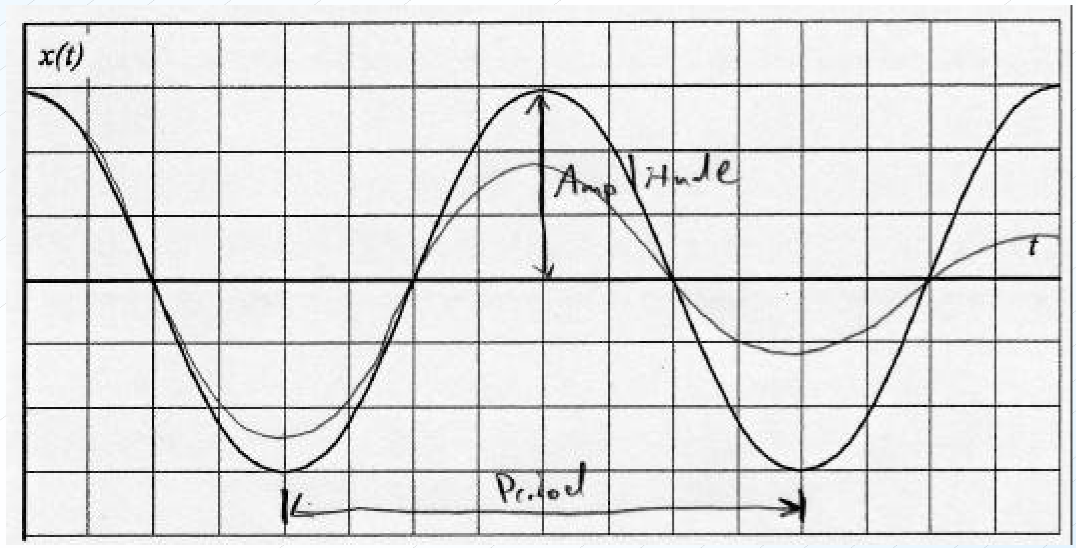
B. Suppose that a retarding force were applied to cause the oscillator to become underdamped.

On the axes above, sketch a qualitatively correct *x* vs. *t* graph for the oscillator when it is released *from rest* at the *same initial position as before*. Explain how you decided to draw the graph the way you did.

Underdamped oscillator pretest: Results

After lecture instruction, GVSU (5 classes) and SPU (1 class)

Most common *incorrect* response (60% - 70%):



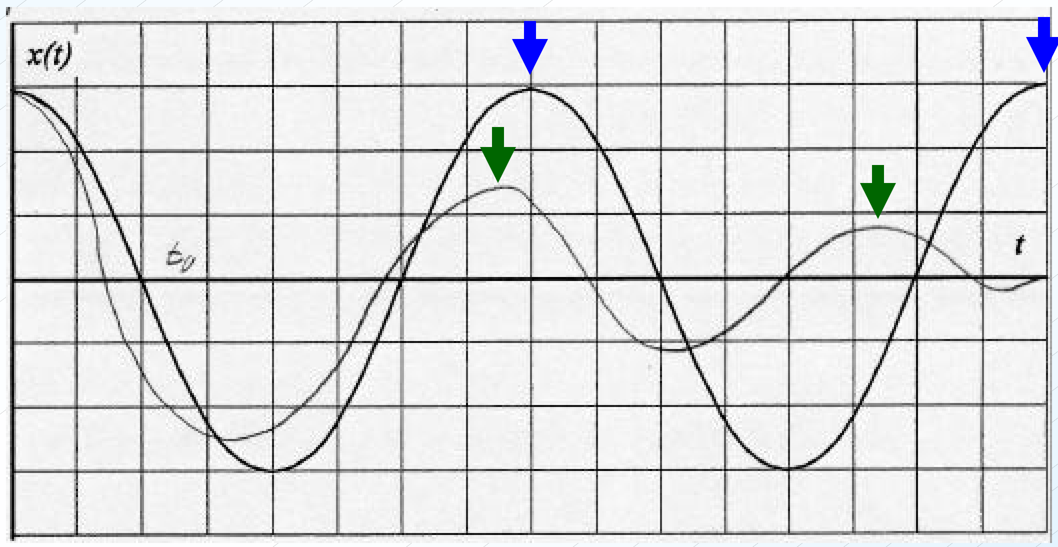
"The amplitude will shrink with time but the period shouldn't change since they are independent of each other."

Failure to recognize that damping force affects frequency

Underdamped oscillator pretest: Results

After lecture instruction, GVSU (5 classes) and SPU (1 class)

Other *incorrect* response:



"The damping agent will prevent the oscillator to reach a certain point in the same amount of time, so the undamped motion 'leads' the damped motion."

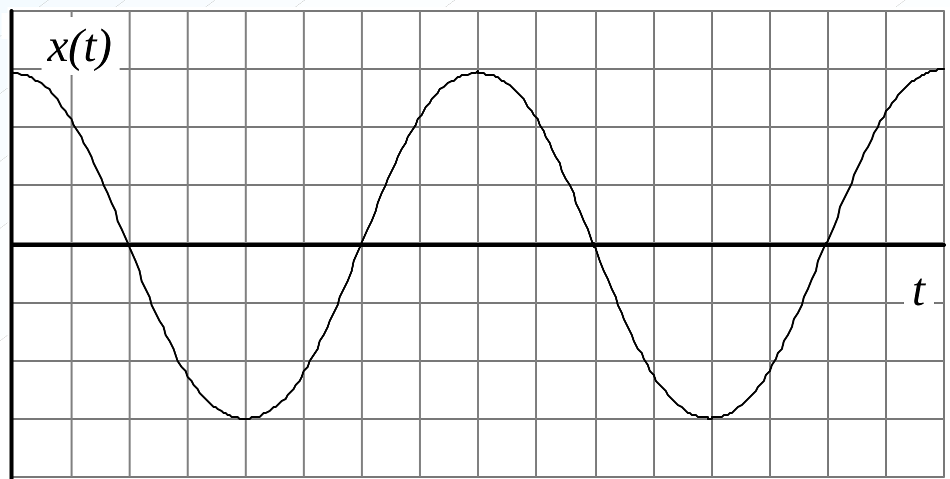
Difficulty connecting verbal and graphical (x vs. t) representations of oscillatory motion

Underdamped oscillator pretest *(cont.)*

Part ii

The x vs. t graph represents the motion of a simple harmonic oscillator that is released from rest at $t = 0$.

A retarding force is applied, causing the oscillator to become underdamped.



ii. Let time t_o represent the instant when the (underdamped) oscillator first passes through $x = 0$.

At $t = t_o$ is the oscillator moving with *increasing speed*, is it moving with *decreasing speed*, or has it *attained maximum speed?* Explain.*

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* Original version asked whether maximum speed was attained *before*, *after*, or *exactly at* $t = t_o$.

Underdamped oscillator pretest: Results

After lecture instruction, GVSU (4 classes) and SPU (1 class)

Correct response (20% - 30%):

From equation of motion $m\ddot{x} = -kx - c\dot{x}$, if $x = 0$ then the only force present is the damping force $-c\dot{x}$, which opposes velocity.

$\therefore$ Damped oscillator moves with *decreasing speed* at $x = 0$.

Most common *incorrect* response (30% - 50%):

Underdamped oscillator attains maximum speed at time $t = t_o$.

Example: "At x = 0, there is no acceleration, speed is at its max [at that location] and not before."

Overgeneralization from the special case of <u>no</u> damping

Summary of student difficulties

- Students often fail to recognize which factors do and do not affect the frequency (or period) of oscillations
  - Incorrect belief that amplitude affects frequency
  - Failure to recognize that damping *does* affect frequency
- Students often experience difficulty connecting concepts and formal representations of oscillatory motion.
  - Motion graphs of 1D oscillators
  - Differential equations of motion

Building students' physical ***and*** mathematical intuitions about damped oscillators

- ***Motion graphs*** tutorial: Students apply **Newton's laws** in order to recognize effect of damping force on:
  - Period of motion (and hence, frequency)
  - Occurrences of maximum speed
- ***Graphical analysis*** tutorial: Students qualitatively analyze **differential equations of motion:**
  - Position (x) and curvature (d^2x/dt^2) for simple harmonic oscillators
  - Time-ordering of turning points, inflection points, and axis crossings for damped oscillators

Topic #2

Conservative force fields

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What we teach **about conservative forces**

in intermediate mechanics

A force $\vec{F}(\vec{r})$ is conservative if and only if:

- the work by that force around any closed path is zero
- the vector curl of the force is zero at all locations
- a potential energy function $U(\vec{r})$ exists so that $\vec{F} = -\vec{\nabla} U$

(generalization of $\vec{E} = -\vec{\nabla} V$ from electrostatics)

"Equipotential map" pretest

Intermediate mechanics, GVSU

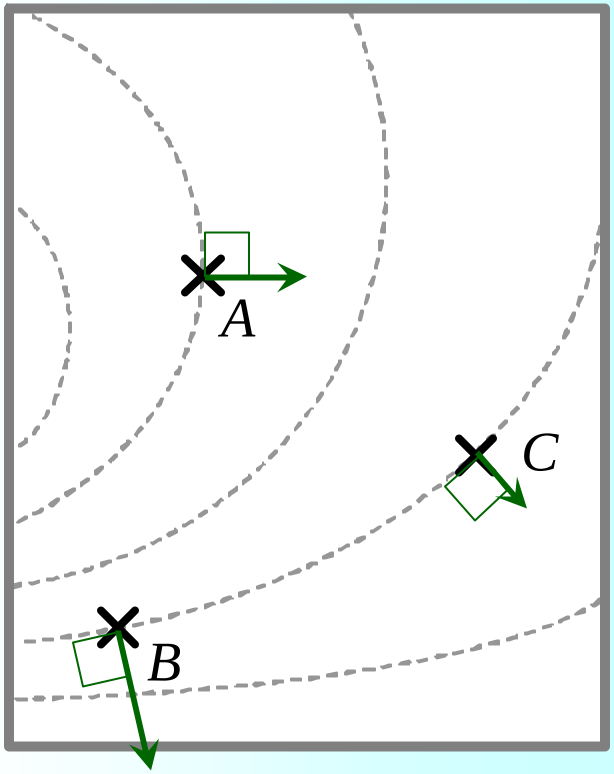
After all lecture instruction in introductory E&M

In the region of space depicted at right, the dashed curves indicate locations of *equal potential energy* for a test charge $+q_{test}$ placed within this region.

It is known that the potential energy at location *A* is *greater than* that at *B* and *C*.

A. At each location, draw an arrow to indicate the direction in which the test charge $+q_{test}$ would move when released from that location. Explain.

B. Rank the locations *A, B,* and *C* according to the magnitude of the force exerted on the test charge $+q_{test}$. Explain your reasoning.



(Qualitatively correct force vectors are shown.)

Equipotential map pretest: Results

Intermediate mechanics, GVSU (N = 43, 6 classes)

After all lecture instruction in introductory E&M

Percent correct *with correct reasoning:*
(rounded to nearest 5%)

Part A (Directions of force vectors)	50%	(22/43)
Part B (Ranking force magnitudes)	20%	(8/43)
Both parts correct	**10%**	**(5/43)**

Similar results have been found among student populations at U. Maine and Seattle Pacific U.

Equipotential map pretest: Results

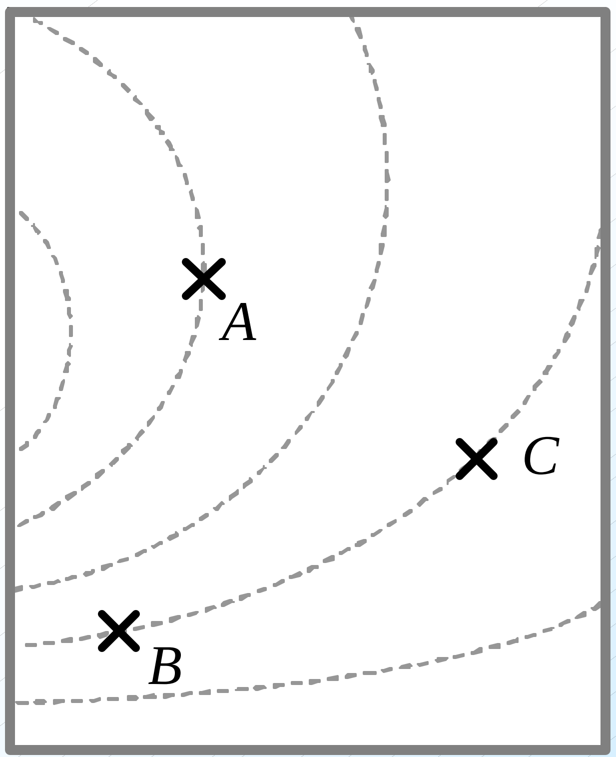
Intermediate mechanics, GVSU

After all lecture instruction in introductory E&M

Most common *incorrect* ranking: $F_A > F_B = F_C$

Example: "The amount of force exerted on q_{test} is dependent on the size of the potential at the point q_{test} is placed."

Example: "Since F is proportional to V, higher V means higher F."



***Failure to discriminate between a quantity** (potential energy U)*
***and its rate of change** (force $\vec{F} = -\vec{\nabla}U$)*

Building students' physical *and* mathematical intuitions about conservative forces

- In Part I of ***Conservative forces and equipotential diagrams,*** students develop:
  - analogy between topographic maps and equipotential diagrams
  - qualitative relationship between local **equipotential lines** and **force vectors** (direction and magnitude)

- In Part II of the tutorial, students:
  - construct **operational definition of gradient** of potential energy
  - motivate quantitative relationship between $\vec{F}$ and $\vec{\nabla} U$

Preliminary conclusions

- Intermediate mechanics students often experience conceptual and reasoning difficulties similar to those identified at the introductory level.
  - Standard lecture instruction, *even in advanced topics*, does not effectively address basic difficulties.
  - Specific difficulties must be addressed *explicitly* and *repeatedly* for meaningful learning.

- An inquiry-based tutorial approach can be effective for both investigating and improving student understanding.
  - Probing student thinking: pretests and classroom observations
  - Addressing difficulties: tutorials and tutorial homework

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