GENERALIZED COORDINATES

In class, we have begun to discuss ways of choosing coordinate systems. We know that you have experience in choosing appropriate coordinates, including the problem we'll discuss below.
This tutorial introduces some formal tools that should match up to your intuitive sense of what's right.

I. Generalized coordinates, intuitively

Consider a simple projectile (no air resistance, constant weight, point particle) shot at a 45-degree angle with an initial speed of v_o. If you were to solve for the particle's maximum height or time of flight, you would break the motion into two coordinates, distance and height, and solve each somewhat independently.

A. Consider a block sliding down straight down an inclined plane.

1. Sketch the system, including the coordinate system you would use to think about the situation.

2. We're guessing you chose a 2-dimensional coordinate system. (If you didn't, answer as if…)

    a. Why didn't You include a third dimension in your coordinate system?

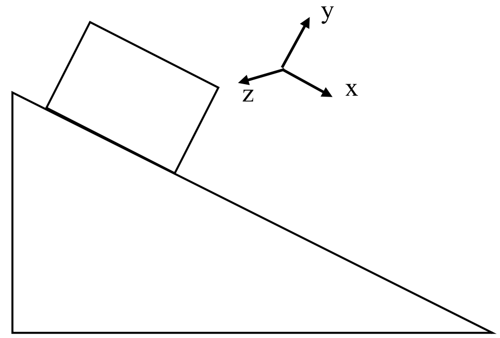
    b. How many coordinates are actually needed to describe the motion of (but not the forces on) the block? Think about the motion as simply as possible. What would you choose for this coordinate?

    c. Why did you include the *second* coordinate? Explain.

No, really, read the following paragraph:
The coordinate you chose to represent the motion of the block is called a *generalized coordinate.*
When you describe a system in terms of generalized coordinates, you pick the coordinates with the goal of completely describing the motion of the system in the fewest number of coordinates. In the Lagrangian formulation, the kinetic and potential energies are written in terms of the generalized coordinates of the system and the equation of motion is found from these. Describing the forces isn't necessary, so you can use fewer coordinates.

II. Generalized coordinates, formally

At right, we show the block from section I with a fixed, 3-D, Cartesian coordinate system where z points out of the page.



In section I, you used your intuitions about the block-and-plane system to figure out that only one coordinate was needed to describe the motion of the system. The formal method for finding the number of generalized coordinates needed is to first find the total number of degrees of freedom of the system and then to subtract the number of constraints on the system.

A. In the block example, how many ways could the block potentially move in space? Ignore the incline for a moment.

B. Obviously, the incline can't be ignored and places a constraint on the motion of the block. Equations of constraint are derived from the physical situation and allow us to remove unnecessary coordinates. They can be due to physical limitations on the motion of objects in the system or can be chosen simply because it is convenient to do so.

1. What constraint does the ramp impose on the block? Describe it in words and write an equation.

2. Are there any other constraints, either physical or convenient? Write an equation for each additional constraint you find.

C. How many generalized coordinates are needed? Does this analysis agree with your informal reasoning of A.2.b? If not, reconcile your answers. By "reconcile," we mean "make consistent" and "come to an understanding about the source of the inconsistency."

D. Consider a new system. Could it have an equal number of degrees of freedom and equations of constraint? More equations of constraint than degrees of freedom? Explain the physical situation in each case.

E. Consider solving systems of linear equations. How are the relationships between the numbers of equations and the numbers of unknowns consistent (or different) from your answer to #4? (discuss with your group).

III. A movable ramp

In the previous example, it was fairly clear how to choose the generalized coordinate – we just took away two coordinates from an already established coordinate system. However, the choice of coordinates is not always so straightforward for more complicated systems.

Think again about the block sliding down the ramp. Now, allow the ramp to move as well.

A. Sketch the system. Describe what will happen to the block and the ramp if the block is released from rest at the top of the ramp. (Describe the motion qualitatively and explain the causes.)

B. Decide how many generalized coordinates are now needed to describe the system. (Hint: Think about each object individually.) Use the intuitive method of Section I or the formal method of Section II. Use both, if possible.

C. It seems reasonable to keep the "down the ramp" coordinate as one of the generalized coordinates. What would you choose for the other coordinate(s)? (Which object have you not yet described?)

D. Generalized coordinates are different from fixed coordinates in several ways.

   1. Do your coordinates have the same origin? If not, describe the origin of each coordinate. Do you know of other coordinate systems with the same property?

   2. Are your coordinates orthogonal? Do you know of other coordinate systems with the same property?

With your group, speculate on why the Newtonian (force) formulation of mechanics uses coordinates that have a common origin and are orthogonal, and the Lagrangian (energy) formulation need not. Discuss your thoughts with an instructor.

