HARMONIC MOTION IN TWO DIMENSIONS

I. Frequencies of motion

A small block of mass *m* is attached to a massless spring (spring #1) with force constant k_1 and placed upon a frictionless horizontal surface. The block undergoes simple harmonic motion in a straight line.

A. If you wished to double the frequency of oscillation (using the same block), (i) would you *increase* or *decrease* the force constant, and (ii) by *what factor* would you change the force constant? Discuss your reasoning with your partners.

Now imagine that another spring (spring #2) with force constant k_2 is attached to the block so that the block can undergo oscillatory motion simultaneously in orthogonal directions.

Note: We will assume that spring #1 is always parallel (or essentially parallel) to the *x*-axis, and that spring #2 is always parallel (or essentially parallel) to the *y*-axis.

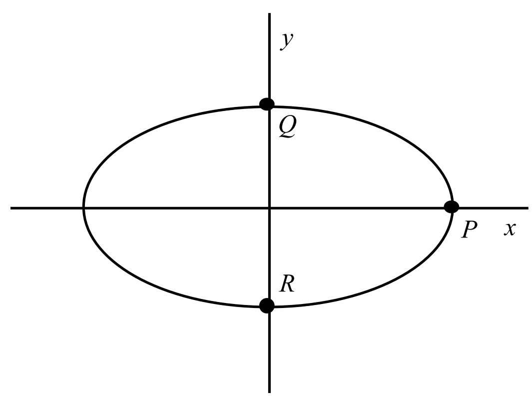
B. If the force constants k_1 and k_2 were equal to each other, how would the frequencies of motion along the *x*- and *y*-axes compare to one another? Explain your reasoning.

   How would the frequency of oscillations along the *x*-axis compare to that along the *y*-axis if instead the force constants were *unequal, e.g.,* if $k_1 > k_2$? Explain.

C. The diagram below right shows the *x*-*y* trajectory of an example 2-D oscillator. The amplitude of motion along the *x*-axis is larger than that along the *y*-axis.

   1. For each period of oscillation along the *x*-axis, how many periods of oscillation occur along the *y*-axis? Explain how you can tell.

   2. For the 2-D oscillator described here in part C, is k_1 *greater than, less than,* or *equal to* k_2? Explain how you can use your results from part B to support your answer.



x-*y* trajectory of a 2-D oscillator

✓ **STOP HERE** and check your results with an instructor.

II. Trajectories of 2-D isotropic oscillators

For the remainder of this tutorial, consider an *isotropic* oscillator, *i.e.,* consider a 2-D oscillator for which the force constants are equal: $k_x = k_y = k$. The positions $x(t)$ and $y(t)$ of the oscillator can be written:

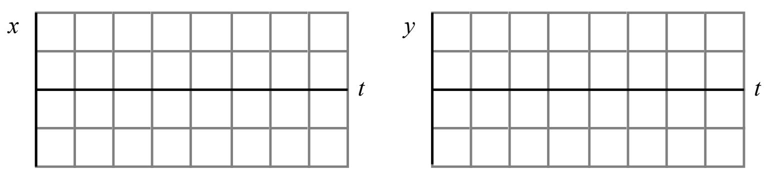
$$x(t) = A_1 \cos(\omega_o t + \varphi); \quad y(t) = A_2 \cos(\omega_o t + \varphi + \delta)$$

A. In the expressions for $x(t)$ and $y(t)$ presented above, explain why *two* phase angles (φ and δ) are needed in order to make those expressions as general as possible.

B. The *x-y* trajectory shown in section I (on the preceding page) can result from many possible initial conditions of motion. For each set of initial conditions listed below, (i) sketch qualitatively correct graphs of $x(t)$ and $y(t)$, and (ii) state the appropriate values of the phase angles φ and δ.

IMPORTANT: Use values for φ and δ within the ranges: $-180° \le \varphi \le 180°$ and $-180° \le \delta \le 180°$.

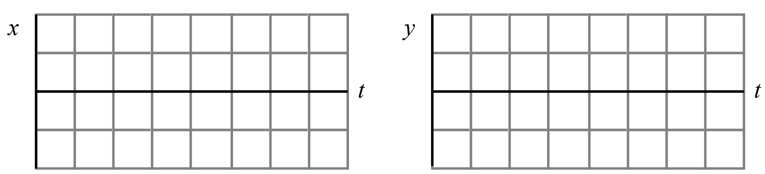
- Initial position at point *P,* initial velocity in +*y* direction



φ =

δ =

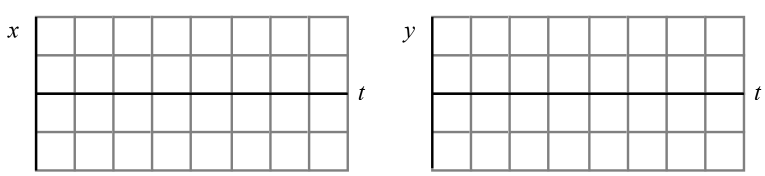
- Initial position at point *P,* initial velocity in –*y* direction



φ =

δ =

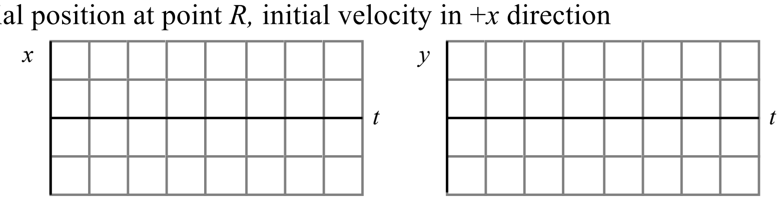
- Initial position at point *Q,* initial velocity in +*x* direction



φ =

δ =

- Initial position at point *R,* initial velocity in +*x* direction



φ =

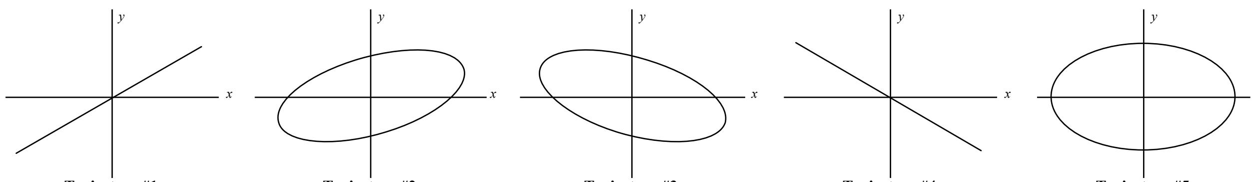
δ =

C. Generalize your results thus far:

- What is the sign of δ if the oscillator follows the trajectory clockwise? Counter-clockwise?
- When an isotropic oscillator in two dimensions follows an elliptical trajectory whose axes coincide with the *x*-*y* coordinate axes, what is the value of $|\delta|$?

✓ **STOP HERE** and check your results with an instructor.

D. Shown below are several possible trajectories for a 2-D isotropic oscillator. Mathematically the only property that makes one trajectory different from another is the value of the phase angle δ.



Trajectory #1 **Trajectory #2** **Trajectory #3** **Trajectory #4** **Trajectory #5**

1. Identify the trajectory that best represents the case in which: (i) $\delta = 0°$, (ii) $\delta = 90°$, (iii) $\delta = 45°$. Explain.

2. Now extend and generalize your results: For each value of δ below, (i) identify the trajectory that best corresponds to that case <u>and</u> (ii) state (if appropriate) whether the oscillator follows that trajectory *clockwise* or *counter-clockwise.* Discuss your reasoning with your partners.

δ = 180°	δ = 135°	δ = 90°
δ = 45°	δ = 0°	δ = –45°
δ = –90°	δ = –135°	δ = –180°

✓ **STOP HERE** and check your results with an instructor.