Homework: Damped oscillations: Energy loss and the quality factor Name ________________________

1. An ideal (frictionless) simple harmonic oscillator is set into motion by releasing it from rest at $x = +\ 0.750$ m.

   The oscillator is set into motion once again from $x = +\ 0.750$ m, except the oscillator now experiences a retarding force that is linear with respect to velocity. As a result, the oscillator does not return to its original starting position, but instead reaches $x = +\ 0.700$ m after one period.

   a. During the first full oscillation of motion, determine the fraction of the oscillator's total energy that was dissipated due to the retarding force. Explain.

   b. Predict the maximum displacement of the oscillator upon completing its *fourth* full oscillation. Explain your reasoning.

   c. Determine the quality factor of the oscillator. Show all work.

2. Consider two identical simple harmonic oscillators (1 and 2) to which are applied different damping forces. The two oscillators become underdamped, with the frequency of the first oscillator now greater than that of the second (*i.e.,* $\omega_{d,1} > \omega_{d,2}$).

   a. Which oscillator has the larger damping constant: *oscillator 1, oscillator 2, neither,* or *is there insufficient information to tell?* Explain your reasoning.

   b. Which oscillator retains the larger fraction of its amplitude with each oscillation: *oscillator 1, oscillator 2, neither,* or *is there insufficient information to tell?* Explain your reasoning.

3. In section II of the tutorial we found the following expression for the quality factor: $Q = \dfrac{2\pi}{1-e^{-2\gamma T_d}}$.

   In this problem we consider the case in which the quantity γT_d is much smaller than one ($\gamma T_d \ll 1$).

   a. Show that such an oscillator is not only underdamped but *weakly damped, i.e.,* that the damping constant is much smaller than the natural frequency of the oscillator: $\gamma \ll \omega_o$.

   b. Extend your results by showing that the quality factor of a weakly damped oscillator can be approximated as $Q \cong \omega_d/2\gamma$, where ω_d is the angular frequency of the oscillator.

      Hint: Use the power series expansion of the exponential function: $e^x = 1 + x + \dfrac{1}{2!}x^2 + \dfrac{1}{3!}x^3 + \ldots$

4. Consider a weakly damped oscillator that is released from rest at $t = 0$ and whose amplitude is smaller by a factor of e after undergoing N full oscillations.

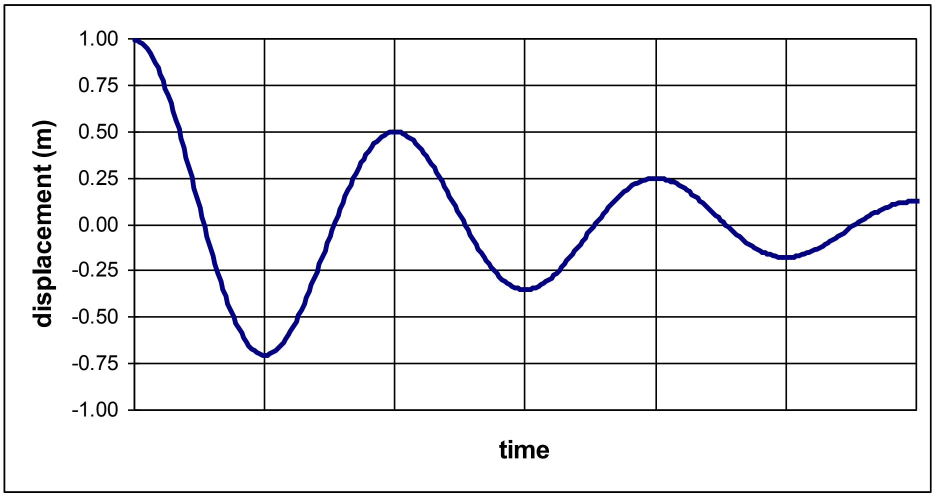
   a. Write an expression for the period of oscillation T_d in terms of N and the damping constant γ. Explain your reasoning. (*Hint:* Recall your interpretation of the quantity $e^{-\gamma T_d}$.)

   b. The quality factor of this oscillator might be *proportional to N, inversely proportional to N,* or *independent of N.* Which of these options makes the most sense? Explain qualitatively, *without* performing any calculations or referring to any formulas.

   c. Verify your answer in part b above finding an expression for Q in terms of N. Show all work.

5. The *x vs. t* graphs in parts a and b below represent the motion of an underdamped oscillator released from rest at $t = 0$.

   a. On the graph below, sketch a qualitatively correct *x. vs. t* graph (drawn to the same scale as the original graph) for another oscillator having the same damping constant γ as the original oscillator but a larger quality factor. Explain how you decided to draw the new graph. (*Hint:* The frequency of new oscillator may be different from that of the original.)



   b. On the graph below, sketch a qualitatively correct *x. vs. t* graph (drawn to the same scale as the original graph) for an oscillator having the same quality factor as the original oscillator but a smaller damping constant γ. Explain how you decided to draw the new graph.