INSTRUCTOR NOTES
Phase space diagrams: Simple harmonic motion

Emphasis

This tutorial introduces students to phase space *($\dot{x}$ vs. x)* diagrams and guides them to construct and interpret such diagrams for simple harmonic motion.

Prerequisites

Students should already have studied simple harmonic motion. Completion of the tutorial *Simple harmonic motion* is not required but is recommended prior to this tutorial. This tutorial may also serve as the first classroom exposure students have to the topic of phase space diagrams.

TUTORIAL PRETEST

The pretest asks students to sketch qualitatively correct phase space diagrams for several different motions, including situations in which the object in question is at rest, moves with constant velocity, moves with constant acceleration, or undergoes simple harmonic motion.

As is the case with any new representation of motion, difficulties often arise as students attempt to interpret phase space diagrams and relate such diagrams to other representations, such as *x vs. t* graphs and $\dot{x}$ *vs. t* graphs. Most students, even before lecture instruction, may recognize that an object at rest (part A of pretest) can be represented by a single dot located along the *x*-axis of a phase space diagram. However, in part B, for the roller skater moving with constant velocity in the negative-*x* direction, many students fail to recognize that the phase space trajectory must be drawn below the *x*-axis rather than above it. Also, many students will attempt to draw sinusoidal (not elliptical) trajectories for a harmonic oscillator (part D).

TUTORIAL SESSION

Equipment and handouts

Each group will need a whiteboard and set of markers, or a large sheet of paper. Each student will need a copy of the tutorial handout (no special handouts required).

Discussion of tutorial worksheet

In parts A – C of the tutorial the students rearrange the equation stating the energy conservation principle for a simple harmonic oscillator in order to show that it can be rewritten in the form of an ellipse equation for *x* and $\dot{x}$. In doing so students interpret the following quantities *a* and *b* in the equation as follows:

$$\frac{x^2}{a^2} + \frac{\dot{x}^2}{b^2} = 1 \quad \Rightarrow \quad a = \sqrt{\frac{2E}{k}} \text{ is the amplitude, } \quad b = \sqrt{\frac{2E}{m}} \text{ is the maximum speed attained.}$$

Students also interpret the ratio *b*/*a* as the angular frequency of oscillation, $\omega = \sqrt{k/m}$.

As students continue to part D, they use their interpretations to draw different phase space trajectories for an oscillator with a given angular frequency but different values of total energy. In particular, for values of energy that differ by a factor of 4, students must recognize that the elliptical trajectories (i) must have the same *b*/*a* ratio (or eccentricity) while at the same time (ii) the corresponding axes differ in size by a factor of 2 (not 4). For students struggling with the latter point, a helpful question might be: "In order to quadruple the maximum potential [or kinetic] energy, how must you change the amplitude [or maximum speed]?"

Students finish the tutorial by considering the possible direction (whether clockwise or counter-clockwise) that is followed along the phase space trajectory as well as the physical reasons for the

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trajectory to cross the x- and $\dot{x}$-axes at right angles. Students who answered part B of the pretest incorrectly (for constant velocity motion in the negative-x direction) may need guidance to recognize that the direction is not arbitrary but instead must be clockwise.

TUTORIAL HOMEWORK

The homework questions give students practice in interpreting and constructing phase space diagrams. Other questions require students to translate between numerical calculations of relevant parameters and quantitatively correct phase space diagrams.

1. Students apply the main ideas developed in the tutorial to a simple harmonic oscillator with known mass, spring constant, and initial conditions.
2. Students consider the effect of changing either the mass or the spring constant (while keeping the total energy the same) on the phase space trajectory of a simple harmonic oscillator. This problem complements the part of the tutorial in which students considered the effect of changing the energy while keeping the mass and spring constant fixed.
3. Students extend their results quantitatively by translating between numerical calculations of relevant parameters and accurately drawn phase space diagrams.

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