PHASE SPACE DIAGRAMS: SIMPLE HARMONIC MOTION

Consider a simple harmonic oscillator with mass m, spring constant k, and total energy E_o.

A. In terms of x, $\dot{x}$, and the given parameters, write an equation that expresses the fact that the total energy (kinetic plus potential) remains constant.

B. Recall that the equation for an ellipse can be expressed in the form: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Give simple interpretations for the quantities a and b in the ellipse equation.

C. Rewrite your equation from part A so that it fits the form of an ellipse equation in terms of x and $\dot{x}$ (instead of x and y).

1. In terms of the given parameters (k, m, and E_o), write down expressions for the quantities that correspond to a and b in the ellipse equation.

   Give simple interpretations for the expressions you found above for a and b.

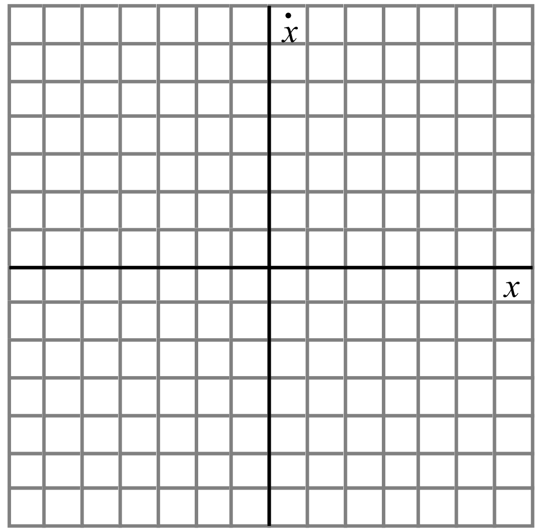
2. Express the ratio (b/a) in terms of the variables given and give a simple interpretation to this quantity.

✓ **STOP HERE** and check your results with an instructor before continuing.

We can illustrate the evolution of physical systems (*e.g.,* a simple harmonic oscillator) by drawing trajectories in *phase space.* In contrast to real three-dimensional space, the dimensions of phase space are velocity $\dot{x}$ and position x along a single axis in real space.

D. Consider a simple harmonic oscillator having an angular frequency 1.5 s^{-1}.

1. On the axes provided, sketch two (elliptical) phase space trajectories for this oscillator, with one trajectory corresponding to a total energy that is *four times* that of the other.



If you were to draw more trajectories for the same oscillator, each corresponding to a different total energy, would any trajectories cross each other? Explain why or why not.

2. In order to correctly show the time evolution of the oscillator, should the trajectories that you have drawn follow *clockwise paths, counter-clockwise paths,* or *does it not matter?*

   Check your answer by considering the motion of the oscillator from a variety of starting points along a given phase space trajectory.

3. For the phase space trajectory of a simple harmonic oscillator, give a *physical* interpretation of the fact that:

   - the trajectory crosses the horizontal *(x)* axis at right angles.

   - the trajectory crosses the vertical $(\dot{x})$ axes at right angles.

