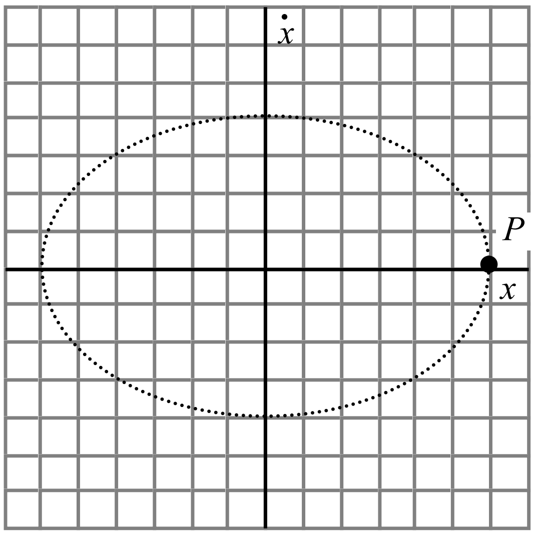
PHASE SPACE DIAGRAMS: DAMPED HARMONIC MOTION

I. Underdamped oscillator

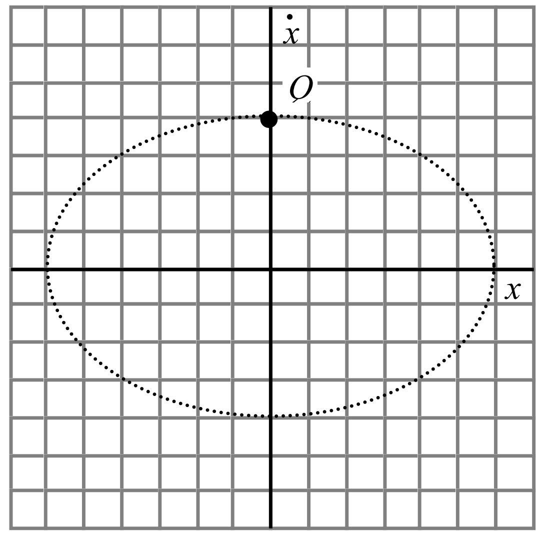
Suppose that a simple harmonic oscillator were subject to a retarding force that is proportional to the velocity of the oscillator.

Each phase space plot shown below corresponds to a motion of the oscillator in the undamped case.

A. On each diagram, sketch the (approximate) phase space trajectory for the situation described under each plot. Discuss your reasoning with your partners.



Case 1: Starts at point *P;* amplitude decreases by a factor of 2 with each oscillation



Case 2: Starts at point *Q;* amplitude decreases by a factor of 4 with each oscillation

B. For a damped oscillator, is it correct for the phase space trajectory to cross the vertical ($\dot{x}$) axis at right angles? Explain why or why not. (*Hint:* In this case, is the net force exerted on the oscillator equal to zero when it passes through $x = 0$?)

For a damped oscillator, is it correct for the phase space trajectory to cross the horizontal *(x)* axis at right angles? Explain why or why not.

C. Are the phase space trajectories that you sketched in part A consistent with your answers in part B? If not, resolve the inconsistencies.

✓ **STOP HERE** and check your results with an instructor before proceeding to the next section.

II. Critically damped oscillator

Now suppose that the oscillator were critically damped, *i.e.,* suppose that the damping factor (γ) for the retarding force were now equal to the angular frequency of the undamped oscillator $(\gamma = \omega_o)$. In this case, the position $x(t)$ of the critically damped oscillator is given by:

$$x(t) = (At + B)e^{-\gamma t}$$

where A and B are arbitrary constants.

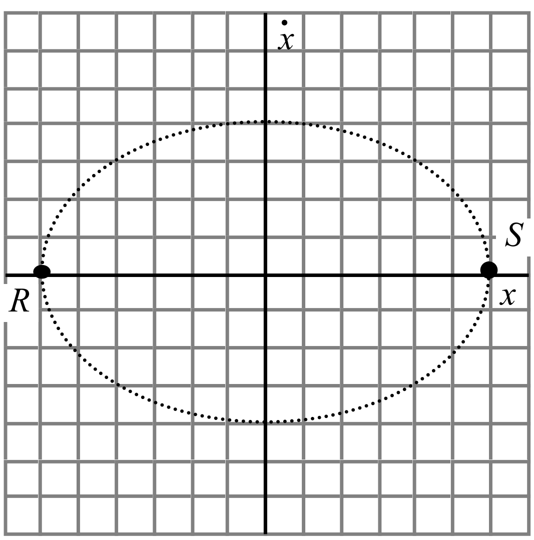
A. Differentiate the above expression for $x(t)$ and show that the parametrized equation $\dot{x}(x, t)$ can be written:

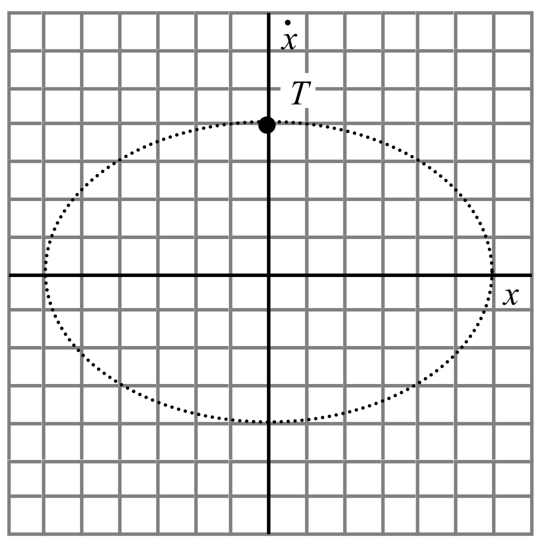
$$\dot{x} = -\gamma x + A e^{-\gamma t}$$

Your result above suggests that the asymptotic behavior (as $t \to \infty$) of the critically damped oscillator can be represented by a *straight line* on a phase space diagram. What is the equation for this line?

B. Each phase space plot shown below corresponds to a motion of the oscillator in the undamped case.

1. On each diagram, *accurately* sketch the line that would describe the asymptotic behavior of the oscillator in the critically damped case. Explain your reasoning.





2. For each of the starting points *(R, S,* and *T)* shown in the diagrams above, draw a qualitatively correct phase space trajectory for the subsequent motion of the critically damped oscillator. Discuss your results with your partners.

✓ **STOP HERE** and check your results with an instructor.

