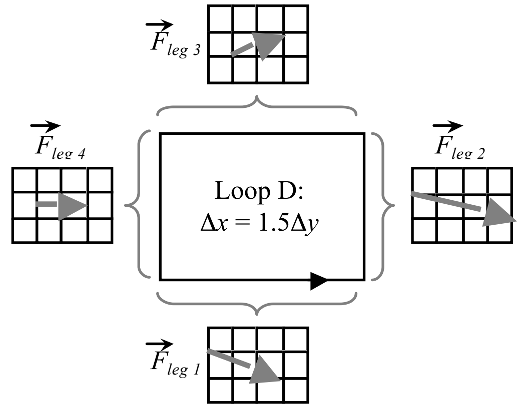
1. Consider the small rectangular loop (loop D) shown at right. The length Δx of the loop is 1.5 times as long as its width Δy.

   Using the same approximations we made in part B of section II of the tutorial (*e.g.,* the force along each leg of the loop may be approximated as the value of the force at the midpoint of that leg), determine whether the work done around loop D is *positive, negative,* or *zero*. Explain your reasoning and show all work.



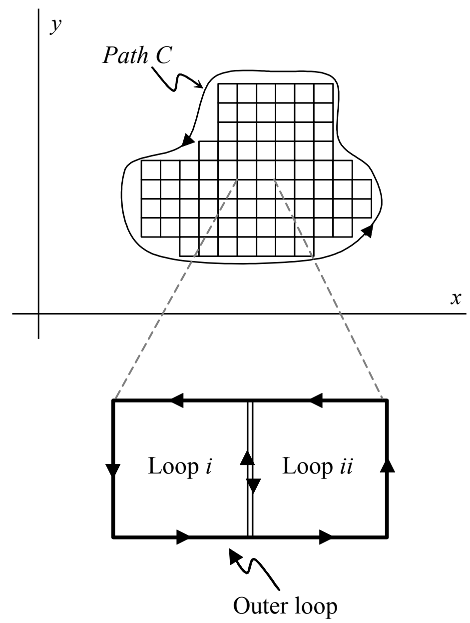
2. In this problem we take care of some "unfinished business" from the tutorial. In particular, we need to extend the results we obtained for a small rectangular loop (from section II of the tutorial) to the case of an arbitrarily shaped closed path in the *x*-*y* plane (such as path *C* shown below right).

   Inside path *C* we can inscribe a large number of tiny closed loops, so that path *C* is (approximated as) the outer perimeter of the tiny loops.

   a. Consider two tiny loops that are adjacent to one another, like loops *i* and *ii* shown at right.

      i. Explain in words why the sum of the works that would be done around both loops is equal in value to the work that would be done around just the outer loop. (*Note:* Do NOT assume that the work around either loop is zero.)

      ii. Extend your reasoning above to explain why works that would be done over all of the tiny loops, added together, would be equal to the work done over the entire closed path *C*.

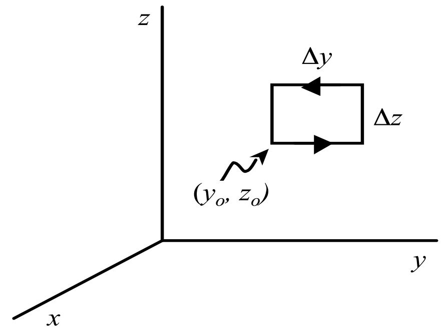


   b. Finally, suppose that it is found mathematically that the curl of the force $\vec{F}$ is equal to zero at every location in the *x*-*y* plane. Explain why this result would imply that work around any path like path *C* must be zero, and thus that the force $\vec{F}$ is conservative.

   c. Recall how you answered the very last question of the tutorial (the one that you might have been able to guess correctly). Was your reasoning (or intuition) correct? If not, how would you change your response that it is complete and correct? Explain.

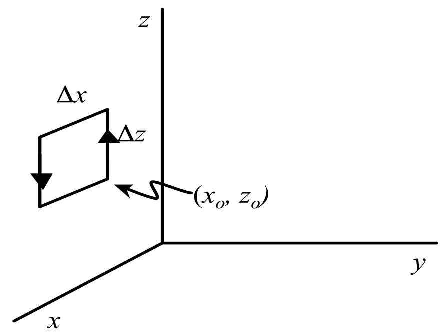
3. In section II of the tutorial you constructed the z-component of the curl of a vector force field, as expressed in Cartesian coordinates. In this problem we construct the *x*- and *y*-components of the curl.

   a. Consider the small rectangular loop of dimensions Δy and Δz in the *y*-*z* plane, shown at right. The corner of the loop nearest the origin has coordinates (y_o, z_o).

   Apply the reasoning you developed in section II of the tutorial in order to find another criterion that a vector force field must satisfy in order to be conservative. In your answer, clearly show *all* steps of your work in constructing (what will become) the *x*-component of the curl of the force.

   b. Repeat part a for the case of the small rectangular loop of dimensions Δx and Δz in the *x*-*z* plane, shown at right. The corner of the loop nearest the origin has coordinates (x_o, z_o). In your answer, clearly show *all* steps of your work in constructing (what will become) the *y*-component of the curl of the force.

4. Consider a force field $\vec{F}(\vec{r}) = \vec{F}(\rho, \varphi, z)$ in **cylindrical** coordinates. Let $F_\rho(\vec{r}) = F_\rho(\rho, \varphi, z)$ and $F_\varphi(\vec{r}) = F_\varphi(\rho, \varphi, z)$ represent the functions that describe the ρ- and ϕ-components of the force.

   In this problem we will extend the reasoning that you developed in section II of the tutorial in order to prove that the *z*-component of the curl of such a force is expressed as:

$$\left(\vec{\nabla} \times \vec{F}\right)_z = \frac{1}{\rho}\left[\frac{\partial}{\partial \rho}\left(\rho F_\varphi\right) - \frac{\partial F_\rho}{\partial \varphi}\right]$$

   a. First, we will need to consider the work done by proceeding around a small loop whose corners have coordinates (ρ, ϕ, z), $(\rho + \Delta\rho, \phi, z)$, $(\rho + \Delta\rho, \phi + \Delta\phi, z)$, and $(\rho, \phi + \Delta\phi, z)$.

   Carefully sketch such a loop, clearly indicating its overall shape as well as the length of each "leg" of the loop. Express each length in terms of ρ, $\Delta\rho$, and $\Delta\phi$. (*Note:* The loop is not rectangular. Two of the four "legs" will have arc-lengths of different sizes.)

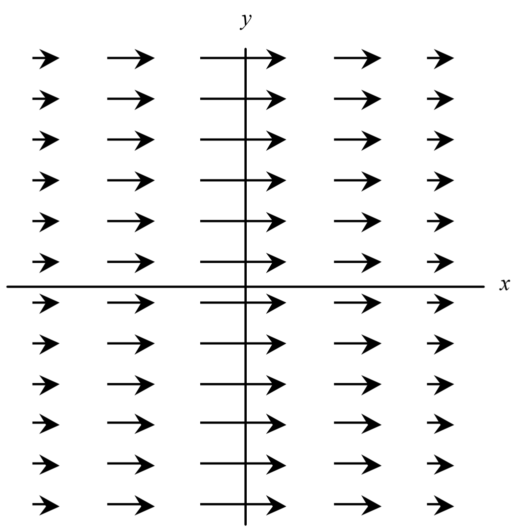
   b. Use your results in part a to help you express the work done around the above loop, and in so doing show that the *z*-component of the curl is correctly expressed as shown above. Show all work.

   Hint: Again, the loop is <u>not</u> rectangular, so think carefully about how to express its area. However, because you will be taking a limit in which the area of the loop tends toward zero, you should find that in this limit your expression for the area will simplify to: area $\approx \Delta\rho\,(\rho\Delta\phi) = \rho\,\Delta\rho\,\Delta\phi$.

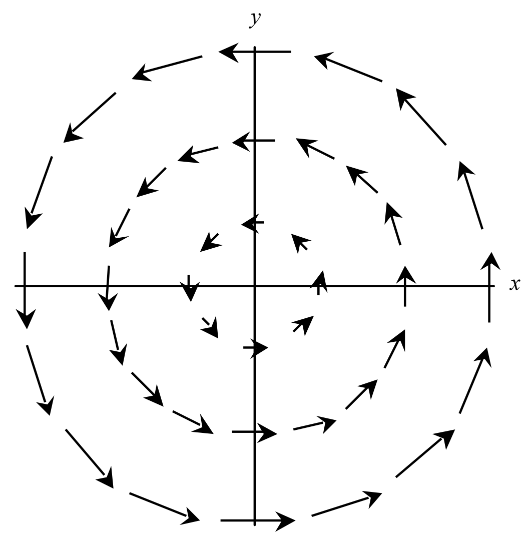
5. Each of the three diagrams on this page represents a force field $\vec{F}(x, y)$ in the *x*-*y* plane.

   For each case, is the vector force field shown a conservative force? Use your results from section II of the tutorial to justify each of your answers.

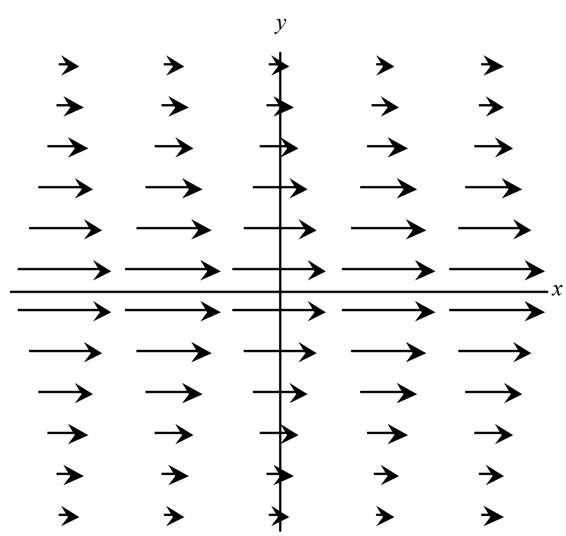
   (*Hint:* For at least one case you can justify your answer by identifying just *one* example of a closed path and considering the value of $\oint \vec{F} \bullet d\vec{r}$ around that path.)



Case #1



Case #2



Case #3

