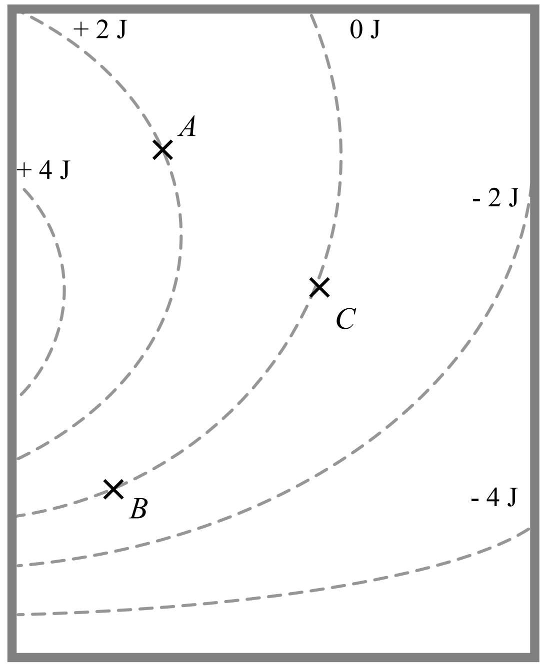
CONSERVATIVE FORCES & EQUIPOTENTIAL DIAGRAMS

I. Equipotential diagrams

The diagram below right represents a region of space near an unknown distribution of charged objects, all fixed in place. The dashed curves indicate positions of *equal* (electrostatic) *potential energy* for a test charge q_{test} that might be placed at various locations within this region. Each curve is labeled according to the appropriate value of potential energy. Three such locations *(A, B,* and *C)* are labeled in this region.



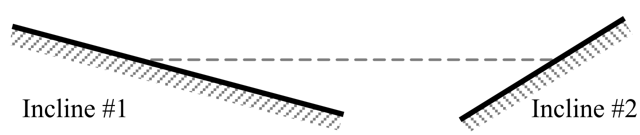
A. Rank the three locations *(A – C)* according to the potential energy of the test charge when placed at those locations. If the potential energy is the same at any locations, state so explicitly.

B. Now rank locations *(A – C)* according to the magnitude of the force that would be exerted on the test charge q_{test} at those locations. If the force is the same magnitude at any locations, state so explicitly. Discuss your reasoning with your partners.

C. Check your ranking for the force magnitudes (part B) by making an analogy between the equipotential map shown above and a *topographic map,* which shows lines of equal elevation. (An instructor can show you and your partners an example topographic map if you like.)

1. Explain how the lines of equal elevation on a topographic map can be used as lines of equal (gravitational) potential energy.

2. What feature of the equal elevation lines indicate the relative *steepness* of the terrain depicted on the map? (*Big hint:* Consider the inclines shown in cross-sectional side view below. On which incline would the elevation lines appear closer to one another?)



SIDE VIEW DIAGRAM

3. Now review your ranking of the force magnitudes from part B (on the preceding page). Do you agree with your ranking? If not, revise your answer and explain how your thinking has changed.

D. Suppose that at each of the labeled locations *(A – C)* the test charge q_{test} is released from rest. At each location on the map, identify the direction in which q_{test} would move when released at that location. Explain how you can tell. (*Hint:* At each location, how would you characterize the direction that best corresponds to "downhill" for the test charge?)

Finally, to summarize your results in this section, draw a vector at each labeled location in order to represent the (net) force exerted on $+q_{\text{test}}$ after it is released at that location. Pay attention to both the *directions* and *relative magnitudes* of your force vectors.

✓ **STOP HERE** and check your results with an instructor before proceeding to the next section.

II. Force and changes in potential energy

We will now overlay a (Cartesian) coordinate system on the equipotential map we used in section I. (See map at the top of the next page.)

A. If the function $U(x, y)$ represents the (electrostatic) potential energy at a location denoted by the coordinates (x, y), explain in words how you could use the map to estimate the value of:

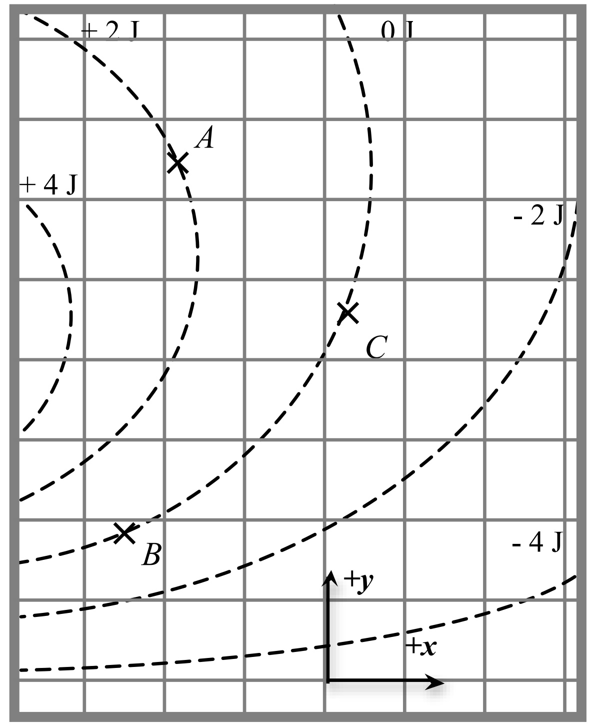
- $\dfrac{\partial U}{\partial x}$ evaluated at a given location (x_o, y_o)

- $\dfrac{\partial U}{\partial y}$ evaluated at a given location (x_o, y_o)

The equipotential map from section I has been reproduced at right with an *x*-*y* coordinate system superimposed on it.

B. For each labeled location *(A – C),* use the equipotential map to answer the following questions.

In the space below, explain how you can use the map to determine the signs and relative sizes of the partial derivatives $\partial U/\partial x$ and $\partial U/\partial y$ at any specified location.



	Location *A*	Location *B*	Location *C*
1. Is $\frac{\partial U}{\partial x}$ *positive, negative,* or *zero?*			
2. Is $\frac{\partial U}{\partial y}$ *positive, negative,* or *zero?*			
3. Which is *greater:* $\left\lvert\frac{\partial U}{\partial x}\right\rvert$ or $\left\lvert\frac{\partial U}{\partial y}\right\rvert$?			

The partial derivatives ($\partial U/\partial x$ and $\partial U/\partial y$) can be thought of as components of a vector called the *gradient* of the potential energy. For a situation in which the potential energy is a function *U(x, y, z)* of all three Cartesian coordinates, the gradient $\vec{\nabla}U$ of the potential energy can be written as follows:

$$\vec{\nabla}U(x,y,z) = \frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}$$

(When the potential energy is a function of only *x* and *y,* as has been the case on this worksheet, we ignore the ($\partial U/\partial z$) term in $\vec{\nabla}U$.)

C. On the basis of your results in part B on the preceding page, carefully draw an arrow on the map at each of the labeled locations *(A, B,* and *C)* to indicate the direction of $\vec{\nabla}U$ at that location.

Summarize your results for the vectors you have drawn:

- Does $\vec{\nabla}U$ point in the direction of *increasing* or *decreasing* potential energy?

- Does $\vec{\nabla}U$ point in a direction in which the potential energy changes *the least* or *the most* with respect to position?

- How does the direction of $\vec{\nabla}U$ compare to the orientation of the equipotential contours near that location?

- Does $\vec{\nabla}U$ have the same magnitude at all locations corresponding to the *same potential energy?* Why or why not?

D. Finally, compare your results for $\vec{\nabla}U$ to the force vectors you drew in section I.

1. Is the force always in the *same direction* as $\vec{\nabla}U$, always in the *opposite direction* from $\vec{\nabla}U$, or *neither?*

2. Is magnitude of the force relatively large at locations where $\vec{\nabla}U$ is *large* or *small* in magnitude?

3. Your results should suggest a mathematical relationship between $\vec{F}$ and $\vec{\nabla}U$. What do you think is that relationship? Explain.

